Prime Numbers

Definition: A *prime number* is an integer $n > 1$ whose only positive divisors are itself and 1.

Example: The first 10 prime numbers are:

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29.$$

Remark: We can make a list of primes by using the method of Erastothenes. (This method is called the Sieve of Erastothenes.)

Theorem 6 (Euclid): There exist infinitely many prime numbers.

Remark: There exist many different proofs of this theorem.

Question: How does the function

$$\pi(x) \ := \ \#\{p \leq x : p \text{ is prime}\}$$

grow as $x \to \infty$?

The answer to this is the Prime Number Theorem:

$$\pi(x) \ \sim \ \frac{x}{\log(x)}, \quad \text{i.e.,} \quad \lim_{x\to\infty} \frac{\pi(x)}{x/\log(x)} \ = \ 1.$$

This was conjectured by Gauss and was proved by Hadamard and by de la Vallée Poussin in 1893.