

The Degree of a Polynomial

Definition: Let

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0 \in F[x],$$

be a non-zero **polynomial** with coefficients in F . If $a_n \neq 0$, then we say that f has **degree n** and write

$$\deg(f) = n.$$

Example: $f(x)$ is a non-zero **constant** polynomial if and only if $\deg(f) = 0$.

Properties: If $f, g \in F[x]$ are non-zero polynomials, then

- (1) $\deg(f + g) \leq \max(\deg(f), \deg(g))$,
- (2) $\deg(f \cdot g) = \deg(f) + \deg(g)$.

Note: In the above equation (1) we **tacitly assumed** that $f + g \neq 0$. We can **avoid** this restriction if we **define** $\deg(0) = -\infty$; then both equations **hold** without any restrictions on f and g .