The Division Algorithm

Theorem 2 (Division Algorithm). Given non-zero integers m and n, there are unique numbers q and r, called the quotient and remainder, respectively, such that

$$m = q \cdot n + r, \tag{1}$$

$$0 \leq r < |n|. \tag{2}$$

Notation. The above quotient q and remainder r are denoted by

$$q = \text{quo}(m, n) \quad \text{and} \quad r = \text{rem}(m, n).$$

Corollary. $n \mid m \Leftrightarrow \text{rem}(m, n) = 0.$

Remarks. 1) The quotient $q = \text{quo}(m, n)$ can also be characterized as the integral part (or floor) of the fraction $\frac{m}{n}$:

$$q = \left[\frac{m}{n}\right] = \text{ the greatest integer } \leq \frac{m}{n}.$$

2) The long division method which you learned in elementary school yields an efficient algorithm for computing the quotient and remainder.

3) In view of the Corollary, the Division Algorithm gives us a quick and effective method for determining whether $n|m$. In other words, we have an effective method for determining whether the equation $nx = m$ has an integer solution or not.