The Euclidean Algorithm

First and Second Version

First Version: By using alternate subtraction:

143	143	91	39	39	26	13
195	52	52	52	13	13	13

Second Version: By using the division algorithm:

$$\begin{aligned} 195 &= 1 \cdot 143 + \boxed{52} \\ 143 &= 2 \cdot \boxed{52} + \boxed{39} \\ \boxed{52} &= 1 \cdot \boxed{39} + 13 \\ \boxed{39} &= 3 \cdot 13 + 0 \end{aligned}$$

Procedure: Given: integers $m, n \neq 0$.

Step 1: Put $r_{-1} = m$, $r_0 = n$.

Step 2: Define successively, for $i = 1, 2, \ldots, k$:

$$r_i = \mathrm{rem}(r_{i-2}, r_{i-1}).$$

Stop when $r_i = 0$. (Thus: $\mathrm{rem}(r_{k-1}, r_k) = 0$.)

Result: $r_k = \gcd(m, n)$.

Example: Find $\gcd(1243, 2147)$.

First Version: By using alternate subtraction:

$$\begin{array}{c||c|c|c|c|c|c} 1243 & 1243 & \boxed{339} & 339 & 339 & \boxed{113} & 113 \\ \hline 2147 & \boxed{904} & 904 & 565 & \boxed{226} & 226 & 113 \end{array}$$

Second Version: By using the division algorithm:

$$\begin{aligned} 2147 &= 1 \cdot 1243 + \boxed{904} \\ 1243 &= 1 \cdot \boxed{904} + \boxed{339} \\ \boxed{904} &= 2 \cdot \boxed{339} + 226 \\ \boxed{339} &= 1 \cdot 226 + 113 \\ 226 &= 2 \cdot 113 + 0 \end{aligned}$$

Thus, by both versions we obtain that

$$\gcd(1243, 2147) = 113.$$