

> Math 211 : MAPLE Homework Solution #2 -- Your NAME

Problem 5: The following procedure uses the second version of the Euclidean algorithm to calculate the gcd $g = \text{gcd}(m,n)$ of two given numbers m and n , and returns g . Furthermore, it displays (on the screen) each step of the Euclidean algorithm.

```
> euclid := proc(m, n) local q, r1, r2, r3;
  r1 := m; r2 := n;
  while r2 ≠ 0 do;
    q := iquo(r1, r2); r3 := irem(r1, r2);
    lprint(r1, '=', q, '·', r2, '+', r3);
    r1 := r2; r2 := r3; od;
  return(r1); end;
euclid := proc(m, n)
  local q, r1, r2, r3;
  r1 := m;
  r2 := n;
  while r2 <> 0 do
    q := iquo(r1, r2); r3 := irem(r1, r2); lprint(r1, '=', q, '*', r2, '+', r3); r1 := r2; r2 := r3
  end do;
  return r1
end proc
```

(1)

Note: On some computer installations, Maple did not accept the backwards quote ('). If this happens, then you are allowed to use the double quote (") instead. (This was mentioned in class.)

(i) Testing the program with your favourite pair of numbers:

```
> euclid(1955, 1949);
1955, '=', 1, '*', 1949, '+', 6
1949, '=', 324, '*', 6, '+', 5
6, '=', 1, '*', 5, '+', 1
5, '=', 5, '*', 1, '+', 0
```

1

(2)

Thus, $\text{gcd}(1955, 1949) = 1$. (In fact, 1949 is a prime number!)

(ii) Testing the program with $m = 11223344556677889977$, $n = 9753124680123456789$

```
> m := 11223344556677889977; n := 9753124680123456789;
      m := 11223344556677889977
      n := 9753124680123456789
```

(3)

```
> euclid(m, n);
11223344556677889977, '=', 1, '*', 9753124680123456789, '+',
1470219876554433188
9753124680123456789, '=', 6, '*', 1470219876554433188, '+',
931805420796857661
1470219876554433188, '=', 1, '*', 931805420796857661, '+',
538414455757575527
```

```

931805420796857661, `=` , 1, `*` , 538414455757575527, `+` ,
393390965039282134
538414455757575527, `=` , 1, `*` , 393390965039282134, `+` ,
145023490718293393
393390965039282134, `=` , 2, `*` , 145023490718293393, `+` ,
103343983602695348
145023490718293393, `=` , 1, `*` , 103343983602695348, `+` ,
41679507115598045
103343983602695348, `=` , 2, `*` , 41679507115598045, `+` ,
19984969371499258
41679507115598045, `=` , 2, `*` , 19984969371499258, `+` ,
1709568372599529
19984969371499258, `=` , 11, `*` , 1709568372599529, `+` ,
1179717272904439
1709568372599529, `=` , 1, `*` , 1179717272904439, `+` ,
529851099695090
1179717272904439, `=` , 2, `*` , 529851099695090, `+` ,
120015073514259
529851099695090, `=` , 4, `*` , 120015073514259, `+` , 49790805638054
120015073514259, `=` , 2, `*` , 49790805638054, `+` , 20433462238151
49790805638054, `=` , 2, `*` , 20433462238151, `+` , 8923881161752
20433462238151, `=` , 2, `*` , 8923881161752, `+` , 2585699914647
8923881161752, `=` , 3, `*` , 2585699914647, `+` , 1166781417811
2585699914647, `=` , 2, `*` , 1166781417811, `+` , 252137079025
1166781417811, `=` , 4, `*` , 252137079025, `+` , 158233101711
252137079025, `=` , 1, `*` , 158233101711, `+` , 93903977314
158233101711, `=` , 1, `*` , 93903977314, `+` , 64329124397
93903977314, `=` , 1, `*` , 64329124397, `+` , 29574852917
64329124397, `=` , 2, `*` , 29574852917, `+` , 5179418563
29574852917, `=` , 5, `*` , 5179418563, `+` , 3677760102
5179418563, `=` , 1, `*` , 3677760102, `+` , 1501658461
3677760102, `=` , 2, `*` , 1501658461, `+` , 674443180
1501658461, `=` , 2, `*` , 674443180, `+` , 152772101
674443180, `=` , 4, `*` , 152772101, `+` , 63354776
152772101, `=` , 2, `*` , 63354776, `+` , 26062549
63354776, `=` , 2, `*` , 26062549, `+` , 11229678
26062549, `=` , 2, `*` , 11229678, `+` , 3603193
11229678, `=` , 3, `*` , 3603193, `+` , 420099
3603193, `=` , 8, `*` , 420099, `+` , 242401
420099, `=` , 1, `*` , 242401, `+` , 177698
242401, `=` , 1, `*` , 177698, `+` , 64703
177698, `=` , 2, `*` , 64703, `+` , 48292
64703, `=` , 1, `*` , 48292, `+` , 16411
48292, `=` , 2, `*` , 16411, `+` , 15470
16411, `=` , 1, `*` , 15470, `+` , 941
15470, `=` , 16, `*` , 941, `+` , 414
941, `=` , 2, `*` , 414, `+` , 113
414, `=` , 3, `*` , 113, `+` , 75
113, `=` , 1, `*` , 75, `+` , 38
75, `=` , 1, `*` , 38, `+` , 37
38, `=` , 1, `*` , 37, `+` , 1
37, `=` , 37, `*` , 1, `+` , 0

```

Thus, the gcd of these two numbers is 1.

>

Note (for Maple experts): By using the string features of MAPLE (and the command printf), one can get cleaner looking output:

```
> euclid2 := proc(m, n) local q, r1, r2, r3, s;  
  r1 := m; r2 := n;  
  while r2 ≠ 0 do;  
    q := iquo(r1, r2); r3 := irem(r1, r2);  
    s := cat(convert(r1, string), "=", convert(q, string));  
    s := cat(s, "*", convert(r2, string), "+", convert(r3, string));  
    printf("%s\n", s);  
    r1 := r2; r2 := r3; od;  
  return(r1); end;
```

```
euclid2 := proc(m, n)
```

```
  local q, r1, r2, r3, s;
```

```
  r1 := m;
```

```
  r2 := n;
```

```
  while r2 <> 0 do
```

```
    q := iquo(r1, r2);
```

```
    r3 := irem(r1, r2);
```

```
    s := cat(convert(r1, string), "=", convert(q, string));
```

```
    s := cat(s, "*", convert(r2, string), "+", convert(r3, string));
```

```
    printf("%s\n", s);
```

```
    r1 := r2;
```

```
    r2 := r3
```

```
  end do;
```

```
  return r1
```

```
end proc
```

Here are the above two examples using the refined printing statements:

```
> euclid2(1955, 1949);  
1955 = 1*1949 + 6  
1949 = 324*6 + 5  
6 = 1*5 + 1  
5 = 5*1 + 0
```

1

```
> euclid2(m,n);
```

```
11223344556677889977 = 1*9753124680123456789 + 1470219876554433188  
9753124680123456789 = 6*1470219876554433188 + 931805420796857661  
1470219876554433188 = 1*931805420796857661 + 5384144557575527  
931805420796857661 = 1*5384144557575527 + 393390965039282134  
5384144557575527 = 1*393390965039282134 + 145023490718293393  
393390965039282134 = 2*145023490718293393 + 103343983602695348  
145023490718293393 = 1*103343983602695348 + 41679507115598045  
103343983602695348 = 2*41679507115598045 + 19984969371499258  
41679507115598045 = 2*19984969371499258 + 1709568372599529
```

(5)

(6)

19984969371499258 = 11*1709568372599529 + 1179717272904439
1709568372599529 = 1*1179717272904439 + 529851099695090
1179717272904439 = 2*529851099695090 + 120015073514259
529851099695090 = 4*120015073514259 + 49790805638054
120015073514259 = 2*49790805638054 + 20433462238151
49790805638054 = 2*20433462238151 + 8923881161752
20433462238151 = 2*8923881161752 + 2585699914647
8923881161752 = 3*2585699914647 + 1166781417811
2585699914647 = 2*1166781417811 + 252137079025
1166781417811 = 4*252137079025 + 158233101711
252137079025 = 1*158233101711 + 93903977314
158233101711 = 1*93903977314 + 64329124397
93903977314 = 1*64329124397 + 29574852917
64329124397 = 2*29574852917 + 5179418563
29574852917 = 5*5179418563 + 3677760102
5179418563 = 1*3677760102 + 1501658461
3677760102 = 2*1501658461 + 674443180
1501658461 = 2*674443180 + 152772101
674443180 = 4*152772101 + 63354776
152772101 = 2*63354776 + 26062549
63354776 = 2*26062549 + 11229678
26062549 = 2*11229678 + 3603193
11229678 = 3*3603193 + 420099
3603193 = 8*420099 + 242401
420099 = 1*242401 + 177698
242401 = 1*177698 + 64703
177698 = 2*64703 + 48292
64703 = 1*48292 + 16411
48292 = 2*16411 + 15470
16411 = 1*15470 + 941
15470 = 16*941 + 414
941 = 2*414 + 113
414 = 3*113 + 75
113 = 1*75 + 38
75 = 1*38 + 37
38 = 1*37 + 1
37 = 37*1 + 0

