

> **Math 211 : MAPLE Homework Solution #6 -- Your NAME**

Problem 6: Using Maple's chrem(..) command to solve simultaneous congruences:

First system:

> $a := [2, 3, 5, 6]; m := [4, 5, 7, 9];$

$a := [2, 3, 5, 6]$

$m := [4, 5, 7, 9]$

(1)

> $x := \text{chrem}(a, m);$

$x := 978$

(2)

Thus, the solution to the simultaneous congruences is $x = 978 \pmod{4 \cdot 5 \cdot 7 \cdot 9}$. This is correct because $\text{rem}(x, 4) = 2$, $\text{rem}(x, 5) = 3$, $\text{rem}(x, 7) = 5$, $\text{rem}(x, 9) = 6$. We can check this by using the command

> $[\text{seq}(\text{irem}(x, m[i]), i = 1 .. 4)];$

$[2, 3, 5, 6]$

(3)

The result is the set of values of a. A better way to check this is by using the evalb command:

> $\text{evalb}([\text{seq}(\text{irem}(x, m[i]), i = 1 .. 4)] = a);$

true

(4)

Note: We could also use modp(..) in place of irem(..).

Second system:

Since this system is not in the form $x = a_1 \pmod{m_1}, \dots, x = a_r \pmod{m_r}$, we first have to convert the system to this form. There are two ways to do this:

First solution: Manually convert each congruence to the desired form:

For example, since $3 \cdot 3 = 1 \pmod{4}$, the equation $3x = 2 \pmod{4}$ is equivalent to $x = 6 \pmod{4}$. Similarly, $2x = 3 \pmod{5}$ is the same as $x = 9 \pmod{5}$, and $4x = 5 \pmod{7}$ becomes $x = 10 \pmod{7}$ and $5x = 6 \pmod{9}$ means $x = 12 \pmod{9}$. Thus, we apply chrem with a replaced by b, where $b = [6, 9, 10, 12]$.

> $b := [6, 9, 10, 12];$

$b := [6, 9, 10, 12]$

(5)

> $y := \text{chrem}(b, m);$

$y := 1074$

(6)

This is the correct solution because if we compute

> $[\text{irem}(3 \cdot y, 4), \text{irem}(2 \cdot y, 5), \text{irem}(4 \cdot y, 7), \text{irem}(5 \cdot y, 9)];$

$[2, 3, 5, 6]$

(7)

then we obtain the coefficient vector on the right hand side of the original second system.

Second solution: use chrem with fractions

For example, dividing both sides of $3x = 2 \pmod{4}$ by 3 yields $x = 2/3 \pmod{4}$, and similarly $2x = 3 \pmod{5}$ is the same as $x = 3/2 \pmod{5}$, etc. Thus:

> $\text{chrem}\left(\left[\frac{2}{3}, \frac{3}{2}, \frac{5}{4}, \frac{6}{5}\right], m\right);$

1074

(8)

gives the same answer as before.

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