Theorem 3 (page 187) can be combined with the cofactor expansions to compute determinants in several other easy cases:

1. If a matrix has two rows or columns equal, then it has zero determinant. We can just subtract one of the two equal rows from the other, producing a matrix with the same determinant and a row or column of zeros:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

2. More generally if one row or column is a multiple of another then the determinant is 0:

$$\begin{vmatrix} 1 & 5 & 2 & -1 \\ 2 & 6 & 4 & 2 \\ 3 & 7 & 6 & 1 \\ 4 & 8 & 8 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 5 & 0 & -1 \\ 2 & 6 & 0 & 2 \\ 3 & 7 & 0 & 1 \\ 4 & 8 & 0 & 0 \end{vmatrix} = 0$$

(subtract two times the first column from the second).

3. Sometimes row and column interchanges can be used to reduce a matrix to upper or lower triangular form:

$$\begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & -1 & -79 & -500 \\ 3 & 799 & -89 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ -500 & -79 & -1 & 0 \\ 0 & -89 & 799 & 3 \end{vmatrix} =$$

(1)(2)(-1)(3)=-6.

(interchange columns 1 and 4, and 2 and 3. Two interchanges multiplies the det by $(-1)^2 = 1$.)

4. If A is $n \times n$ then $\det(rA) = r^n \det A$. We have to bring a factor r out of each row. Example:

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$. Then

$$\det(2A) = \begin{vmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 20 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 & 3 \\ 8 & 10 & 12 \\ 14 & 16 & 20 \end{vmatrix} = 2^2\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 14 & 16 & 20 \end{vmatrix} =$$

$$2^3\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{vmatrix} = 8 \det A.$$

5. Be especially careful with r=-1 in (4). (-1)A=-A so $\det(-A) = (-1)^n \det(A)$. Thus $\det(-A)$ is equal to $\det(A)$ if n is even, and to $-\det(A)$ if n is odd.