

# Linear Algebra Commands in MAPLE

## Description of commands in the *LinearAlgebra* Package

Linear Algebra	Description
1. General	
with(LinearAlgebra): type( <i>expr</i> , Matrix)	read in the linear algebra package gives <i>true</i> if <i>expr</i> has the form of a matrix, else false
type( <i>expr</i> , Vector)	gives <i>true</i> if <i>expr</i> has the form of a vector
RowDimension( <i>a</i> )	the number of rows of the matrix <i>a</i>
ColumnDimension( <i>a</i> )	the number of columns of <i>a</i>
Dimensions( <i>a</i> )	the number of rows and columns
Dimension( <i>v</i> )	the dimension of a vector
2. Constructing matrices and vectors	
Matrix([[ <i>a</i> <sub>11</sub> , ..., <i>a</i> <sub>1<i>n</i></sub> ], ..., [ <i>a</i> <sub><i>m</i>1</sub> , ..., <i>a</i> <sub><i>m</i><i>n</i></sub> ]])	build an $m \times n$ matrix $a = (a_{ij})$ by listing its elements row by row
<< <i>a</i> <sub>11</sub> , ..., <i>a</i> <sub><i>m</i>1</sub> >   ...   < <i>a</i> <sub>1<i>n</i></sub> ... <i>a</i> <sub><i>m</i><i>n</i></sub> >>	build an $m \times n$ matrix $a = (a_{ij})$ by listing its elements column by column
Matrix( <i>m</i> , <i>n</i> , <i>f</i> )	build an $m \times n$ matrix whose <i>ij</i> -th entry is $f(i, j)$
Matrix( <i>m</i> , <i>n</i> , ( <i>i</i> , <i>j</i> ) → <i>expr</i> )	build an $m \times n$ matrix with <i>ij</i> -th entry $expr(i, j)$
Vector([ <i>a</i> <sub>1</sub> , ..., <i>a</i> <sub><i>n</i></sub> ]) or < <i>a</i> <sub>1</sub> , ..., <i>a</i> <sub><i>n</i></sub> >	construct a (column) vector with entries $a_1, \dots, a_n$
3. Basic matrix operations	
<i>a</i> + <i>b</i>	the sum of the matrices <i>a</i> and <i>b</i>
<i>c</i> * <i>a</i>	multiplying the matrix <i>a</i> by a scalar <i>c</i>
<i>a</i> . <i>b</i> or Multiply( <i>a</i> , <i>b</i> )	matrix multiplication
<i>a</i> ^ <i>n</i>	the <i>n</i> -th power of a matrix
1/ <i>a</i> or <i>a</i> ^ (-1)	the inverse of a matrix
map( <i>f</i> , <i>a</i> )	the matrix obtained by applying the function <i>f</i> to each entry of <i>a</i>
Transpose( <i>a</i> ) or <i>a</i> ^ %T	the transpose of <i>a</i>
Determinant( <i>a</i> )	the determinant of <i>a</i>
Trace( <i>a</i> )	the trace of <i>a</i>
Rank( <i>a</i> )	the rank of <i>a</i>
Adjoint( <i>a</i> )	the adjoint matrix (matrix of minors) of <i>a</i>

4. Special matrices	
ZeroMatrix( $m, n$ )	the $m \times n$ zero matrix
DiagonalMatrix( $L$ )	generate a diagonal matrix with the list $L$ as the diagonal entries
IdentityMatrix( $n$ )	generate an $n \times n$ identity matrix
VandermondeMatrix( $L$ )	generate a Vandermonde matrix whose 2nd column is the list $L$
BandMatrix( $L$ )	create a tri-diagonal matrix (or an arbitrary band matrix) using the list $L$
JordanBlockMatrix( $[[c, n]]$ )	generate an $n \times n$ Jordan block with eigenvalue $c$
JordanBlockMatrix( $[[c_1, n_1], \dots, [c_r, n_r]]$ )	generate a Jordan matrix with Jordan blocks $(c_1, n_1), \dots, (c_r, n_r)$
CompanionMatrix( $p, x$ )	generate the $n \times n$ companion matrix associated to a monic polynomial $p(x)$ of degree $n$
5. Extracting parts of a matrix	
$a[i, j]$	the $(i, j)$ -th entry of the matrix $a$
Row( $a, i$ )	the $i$ -th row of the matrix $a$
Column( $a, j$ )	the $j$ -th column of the matrix $a$
SubMatrix( $a, [i_1, \dots, i_r], [j_1, \dots, j_s]$ ) or $a[[i_1, \dots, i_r], [j_1, \dots, j_s]]$	the $r \times s$ submatrix of $a$ with row indices $i_k$ and column indices $j_k$
SubMatrix( $a, i_0..i_1, j_0..j_1$ ) or $a[i_0..i_1, j_0..j_1]$	the submatrix of $a$ having row and column indices from $i_0$ to $i_1$ and $j_0$ to $j_1$ , respectively
6. Pasting and altering matrices	
$\langle a, b, \dots \rangle$	join two (or more) matrices $a, b, \dots$ vertically
$\langle a   b   \dots \rangle$	join two (or more) matrices horizontally
Matrix( $m + r, n + s, [a], \text{fill} = c$ );	enlarge the $m \times n$ matrix $a$ by $r$ additional rows and $s$ additional columns with the value $c$
Matrix( $m, n, L$ )	partition a list $L$ of $mn$ elements into an $m \times n$ matrix
DiagonalMatrix( $[a_1, a_2, \dots]$ )	construct a block diagonal matrix using the (square) matrices $a_1, a_2, \dots$
$a[i, j] := \text{expr}$	replace the $(i, j)$ -th entry of matrix $a$ by the expression $\text{expr}$
DeleteRow( $a, i_1..i_2$ )	delete rows $i_1$ to $i_2$ of $a$
DeleteColumn( $a, i_1..i_2$ )	delete columns $j_1$ to $j_2$ of $a$

7. Row and column operations	
RowOperation( $a, [i_2, i_1], c$ )	add $c$ times row $i_1$ to row $i_2$ (thus, the result is put in row $i_2$ )
ColumnOperation( $a, [j_2, j_1], c$ )	add $c$ times column $j_1$ to col. $j_2$
RowOperation( $a, i, c$ )	multiply row $i$ by $c$
ColumnOperation( $a, j, c$ )	multiply column $j$ by $c$
RowOperation( $a, [i_1, i_2]$ )	interchange rows $i_1$ and $i_2$
ColumnOperation( $a, [j_1, j_2]$ )	interchange columns $j_1$ and $j_2$
8. Row reduction and solutions of linear systems	
LinearSolve( $a, b$ )	find the (general) solution of the matrix equation $ax = b$
NullSpace( $a$ )	compute a basis for the nullspace of a matrix $a$
GaussianElimination( $a$ )	find an upper triangular matrix which is row equivalent to $a$
BackwardsSubstitution( $a, b$ )	solve $ax = b$ by back-substitution (if $a$ is upper triangular)
ReducedRowEchelonForm( $a$ )	compute the reduced row echelon form of $a$
HermiteForm( $a$ )	row reduce the integer matrix $a$ to an upper- $\Delta$ integer matrix
SmithForm( $a$ )	row reduce and column reduce the integer matrix $a$ to a diagonal integer matrix
9. Eigenvalues, Eigenvectors	
CharacteristicMatrix( $a, x$ )	compute the characteristic matrix $Ix - a$ ( $x$ a variable)
CharacteristicPolynomial( $a, x$ )	find the characteristic polynomial of $a$
MinimalPolynomial( $a, x$ )	find the minimal polynomial of $a$
Eigenvalues( $a$ )	compute the eigenvalues of $a$
Eigenvectors( $a$ )	find the eigenvectors of $a$
JordanForm( $a$ ) or JordanForm( $a$ , output = [' $J$ ', ' $Q$ ']);	compute the Jordan canonical form $J$ of $a$ and the matrix $q$ such that $a = q^{-1}Jq$
FrobeniusForm( $a$ ) or FrobeniusForm( $a$ , output = [' $F$ ', ' $Q$ '])	compute the Frobenius (or rational) canonical form $F$ of $a$ ; find $p$ such that $a = pFp^{-1}$
IsSimilar( $a, b$ ) or IsSimilar( $a, b$ , output = ['query', ' $C$ '])	determine whether $a$ is similar to $b$ ; if so, find the matrix $q$ such that $a = q^{-1}bq$

10. Vector operations	
convert( $\langle v, w \rangle$ , Vector) $v + w$ $c * v$ $a . v$ Transpose( $v$ ) . $a$ $v . w$ Norm( $v, 2$ ) Normalize( $v$ ) VectorAngle( $v, w$ )	direct sum of two vectors $v, w$ add two vectors $v, w$ multiply the vector $v$ by scalar $c$ multiply the matrix $a$ by the (column) vector $v$ (on the right) multiply the matrix $a$ by the vector $v$ (on the left) the dot (scalar) product of $v$ and $w$ the length or norm $\ v\ $ of a vector $v$ divide the vector $v$ by its length the angle between the vectors $v$ and $w$
11. Vector spaces	
Basis( $L$ ) IntersectionBasis( $[L_1, L_2 \dots]$ ) SumBasis( $[L_1, L_2 \dots]$ ) RowSpace( $a$ ), ColumnSpace( $a$ ) NullSpace( $a$ ) GramSchmidt( $[v_1, \dots, v_n]$ )	find a basis of the vector space spanned by list $L$ of vectors find a basis of the intersection of the spaces spanned by the lists $L_1, L_2, \dots$ find a basis of the sum/union of the spaces spanned by the lists $L_1, L_2, \dots$ find a basis for the row/column space of the matrix $a$ compute a basis for the nullspace of a matrix $a$ apply the Gram-Schmidt procedure to the vectors $v_1, \dots, v_n$
12. Miscellaneous	
IsOrthogonal( $a$ ) LeastSquares( $a, b$ ) Equal( $a, b$ ) convert(evalm(subs( $x = a, f$ )), Matrix);	test whether $a$ is an orthogonal matrix find $x$ such that $\ ax - b\ $ is minimal test whether matrices $a$ and $b$ are equal evaluate the matrix polynomial $f(a)$ (where $f(x)$ is a polynomial)