Review: Dimensions of Linear Sets

Recall: If $V \subset \mathbb{R}^n$ is a linear space, then its dimension is $\dim(V) = |\mathcal{B}|$, where $\mathcal{B}$ is any basis of V.

If $S \subset \mathbb{R}^n$ is a linear set, then $S = \vec{a} + V$, for some/any $\vec{a} \in S$, and a (unique) subspace $V \subset \mathbb{R}^n$. Its dimension is $\dim(S) := \dim(V)$.

Computations: 1) If $V = \langle \vec{v}_1, \ldots, \vec{v}_k \rangle \subset \mathbb{R}^n$ is a linear space (spanned by $\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{R}^n$), put

$$A = (\vec{v}_1 | \ldots | \vec{v}_k), \quad \text{so} \quad V = \mathrm{Colsp}(A)$$

is the column space of A. Then

$$\dim(V) = \mathrm{rank}(A),$$

and a basis of V can be found by using the recipe given in class (using a row echelon form R of A).

2) If $S = \mathrm{Solsp}(A\vec{x} = \vec{b}) \subset \mathbb{R}^n$ is the linear set defined by the system of equations $A\vec{x} = \vec{b}$, then

$$\dim(S) = n - \mathrm{rank(A)},$$

and a vector equation for S can be found by row reduction and back-substitution.

Note that the number d of "free parameters" $t_1, \ldots, t_d$ in the vector equation of S (obtained by this method) equals the dimension of S, i.e., $d = \dim(S)$.