Limits of Matrix Sequences

Definition: Let $A_1, A_2, \ldots$ be a sequence of $m \times n$ matrices, and write

$$A_k = \begin{pmatrix} a_{11}^{(k)} & \ldots & a_{1n}^{(k)} \\ \vdots & & \vdots \\ a_{m1}^{(k)} & \ldots & a_{mn}^{(k)} \end{pmatrix}.$$

We say that the *limit* A of this sequence *exists* and write

$$\lim_{k\to\infty} A_k = A,$$

if $A = (a_{ij})$ is an $m \times n$ matrix such that

$$\lim_{k\to\infty} a_{ij}^{(k)} = a_{ij}, \quad \text{for all } 1 \le i \le m, 1 \le j \le n.$$

Theorem 3 (Rules for limits): Suppose that $A = \lim_{k\to\infty} A_k$ exists and that B, C are matrices such that the matrix product CA_kB is defined. Then:

(a) $\lim_{k\to\infty} A_kB = AB.$

(b) $\lim_{k\to\infty} CA_k = CA.$

(c) $\lim_{k\to\infty} CA_kB = CAB.$