Matrix Polynomials

Notation: Let $A \in M_m(\mathbb{C})$ = set of all $m \times m$ matrices with entries in $\mathbb{C}$ and let

$$f(t) = c_0 + c_1 t + c_2 t^2 + \ldots + c_n t^n \in \mathbb{C}[t]$$

be a complex polynomial. Then $f(A)$ denotes the matrix expression

$$f(A) = c_0 I + c_1 A + c_2 A^2 + \ldots + c_n A^n \in M_m(\mathbb{C}).$$

Theorem 1: If

$$A = \text{Diag}\,(a_1, a_2, \ldots, a_m) = \begin{pmatrix} a_1 & 0 & \ldots & 0 \\ 0 & a_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & a_m \end{pmatrix}$$

is a diagonal matrix, then for any $f(t) \in \mathbb{C}[t]$ we have:

$$f(A) = \text{Diag}(f(a_1), f(a_2), \ldots, f(a_m)).$$

Definition: Two matrices A and B are called similar if $B = P^{-1}AP$ for some invertible matrix P.

Theorem 2: Let A and $B = P^{-1}AP$ be two similar matrices. Then:

$$A^n = PB^nP^{-1}, \quad \text{for all } n \geq 0,$$
$$f(A) = Pf(B)P^{-1}, \quad \text{for all } f \in \mathbb{C}[t].$$