Review of Diagonalization

Definition: 1) $\lambda \in \mathbb{C}$ is an eigenvalue of an $m \times m$ matrix A if $A\vec{v} = \lambda\vec{v}$ for some $\vec{v} \neq \vec{0}$.

2) $\vec{v} \in \mathbb{C}^m$ is an eigenvector of A with respect to $\lambda \in \mathbb{C}$ if $A\vec{v} = \lambda\vec{v}$.

3) The set $E_A(\lambda) = \{\vec{v} \in \mathbb{C}^m : A\vec{v} = \lambda\vec{v}\}$ of eigenvectors w.r.t. λ is called the λ-eigenspace of A.

4) The characteristic polynomial of A is

$$\begin{aligned} \mathrm{ch}_A(t) &= \det(tI - A) = (-1)^m \det(A - tI) \\ &= t^m + a_1 t^{m-1} + \ldots + a_m. \end{aligned}$$

Facts: 1) $\lambda \in \mathbb{C}$ is an eigenvalue of $A \Leftrightarrow \mathrm{ch}_A(\lambda) = 0$.
Thus: A has precisely m eigenvalues (w. mult.'s).

2) If A has m distinct e.val's, then A is diagonable.

3) A is diagonable if and only if $\mathbb{C}^m$ has a basis consisting of eigenvectors of A.

4) If $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m \in \mathbb{C}^m$ is a basis of eigenvectors (so $A\vec{v}_1 = \lambda_1\vec{v}_1, A\vec{v}_2 = \lambda_2\vec{v}_2, \ldots$), then

$$A \underbrace{(\vec{v}_1|\vec{v}_2|\ldots|\vec{v}_m)}_{P} = (\vec{v}_1|\vec{v}_2|\ldots|\vec{v}_m)\mathrm{Diag}(\lambda_1, \ldots, \lambda_m),$$

or, equivalently: $A = P\mathrm{Diag}(\lambda_1, \ldots, \lambda_m)P^{-1}$.