Evaluating Matrix Polynomials: Method I (Diagonable Case)

Given: an $m \times m$ matrix A:

1) Verify that A is diagonable by checking whether any one of the following criteria applies:

- Is A a real symmetric matrix?
- Does A have m distinct eigenvalues? (Factor the characteristic polynomial $\mathrm{ch}_A(t)$.)
- Do the eigenvectors of A form a basis of $\mathbb{C}^m$?

Note: If none of the above criteria apply, then A is not diagonable and this method cannot be used.

2) Compute the eigenvalues and the corresponding eigenspaces of A and hence find a basis of $\mathbb{C}^m$ consisting of eigenvectors $\vec{v}_1, \ldots, \vec{v}_m$ of A, with corresponding eigenvalues $\lambda_1, \ldots, \lambda_m$. Then the matrix $P = (\vec{v}_1|\ldots|\vec{v}_m)$ is invertible and has the property that $A = PDP^{-1}$, where $D = \mathrm{Diag}(\lambda_1, \lambda_2, \ldots, \lambda_m)$.

3) Use the following formula (cf. Theorem 2, Corollary) to calculate $f(A)$:

$$f(A) = P\mathrm{Diag}(f(\lambda_1), f(\lambda_2), \ldots, f(\lambda_m))P^{-1}.$$