Jordan Blocks

Definition: Given $\lambda \in \mathbb{C}$ and $m \in \mathbb{N}$, the $m \times m$ matrix

$$J(\lambda, m) = \begin{pmatrix} \lambda & 1 & 0 & \cdots & \cdots & 0 \\ 0 & \lambda & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & 0 \\ \vdots & & & \ddots & \lambda & 1 \\ 0 & \cdots & \cdots & \cdots & 0 & \lambda \end{pmatrix}$$

is called the Jordan block of size $m \times m$ associated to $\lambda \in \mathbb{C}$.

Notes: 1) We have

$$\mathrm{ch}_{J(\lambda,m)}(t) = (t - \lambda)^m \text{ and } E_{J(\lambda,m)} = \{c \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} : c \in \mathbb{C}\}.$$

Thus, $J(\lambda, m)$ is not diagonable (if $m \geq 2$).

2) The matrices $J(\lambda, m)$ are the "building blocks" of all non-diagonable matrices (up to similarity).

3) The only obstruction to diagonalizing a matrix is the presence (up to similarity) of Jordan blocks.

4) Every matrix can be put into a "generalized diagonal form", called the Jordan canonical form. This is the closest that a matrix can come to being diagonable.