The Jordan Canonical Form

Definition: A block diagonal matrix

$$J = \text{Diag}(J_1, J_2, \ldots, J_r)$$

determined by Jordan blocks $J_i = J(\lambda_i, m_i)$ is called a Jordan matrix.

Theorem 5: Every square matrix A is similar to a Jordan matrix. In other words, there is an invertible matrix P such that

$$P^{-1}AP = \text{Diag}(J_1, \ldots, J_i, \ldots)$$

is a block diagonal matrix consisting of certain Jordan blocks $J_i = J(\lambda_i, k_i)$ associated to the eigenvalues of A. These blocks are uniquely determined by A (up to order).

Historical Remark: The terms Jordan block, Jordan matrix etc. honour Camille Jordan (1838 – 1922), who in 1870 published the influential book "Traité des substitutions et des équations algébriques" (667 pages). Van der Waerden (in his "History of Algebra") calls this book "a masterpiece of mathematical architecture". The above theorem is found (in essence) in book 2, chapter 3 of Jordan's Traité.