Evaluating Matrix Polynomials: Method I

Method I: (Non-diagonable case) Given a matrix A:

1) Find the eigenvalues λ_i, the Jordan blocks $J_i = J(\lambda_i, m_i)$ and hence the associated Jordan matrix $J = \mathrm{Diag}(J_1, J_2, \ldots, J_r)$.
2) Find a matrix P such that $A = PJP^{-1}$.
3) Use Theorem 7 below to calculate $f(J_i)$.
4) Apply Theorem 6 to calculate $f(A)$:

$$f(A) = P\mathrm{Diag}(f(J_1), f(J_2), \ldots, f(J_r))P^{-1}.$$

Theorem 7: Let $J = J(\lambda, m)$ be a Jordan block. Then for any $f \in \mathbb{C}[x]$:

$$f(J) = \begin{pmatrix} f(\lambda) & f'(\lambda) & \frac{1}{2}f''(\lambda) & \ldots & \frac{1}{(m-1)!}f^{(m-1)}(\lambda) \\ 0 & f(\lambda) & f'(\lambda) & \ldots & \frac{1}{(m-2)!}f^{(m-2)}(\lambda) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & f(\lambda) & f'(\lambda) \\ 0 & \ldots & \ldots & 0 & f(\lambda) \end{pmatrix}$$