Evaluating Matrix Polynomials: Method II

Theorem 8 (Cayley-Hamilton). For any square matrix A we have:

$$\mathrm{ch}_A(A) = 0.$$

Historical Notes:

William Rowan Hamilton (1805 - 1865): invented in 1843 the quaternions (or Hamiltonians) $\mathbb{H}$.

Arthur Cayley (1821 – 1895): invented in 1845 the octonians (or Cayley numbers) $\mathbb{O}$.

Note: We have the hierarchy: $\mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{O}$.

Method II: (of evaluating matrix polynomials)

Given: an $m \times m$ matrix A and a polynomial $f(t)$.

Compute: $f(A)$.

1) Calculate the characteristic polynomial $\mathrm{ch}_A(t)$.

2) Compute $r(t) = \mathrm{rem}(f, \mathrm{ch}_A)$. [Use long division or the substitution method.]

3) Then $f(A) = r(A)$, where $r(A)$ is evaluated the naive way.