The Generalized Remainder Formula

Theorem 9 (The Generalized Substitution Method):
Given polynomials $f(t)$ and

(1) $$g(t) = (t - \lambda_1)^{m_1} \dots (t - \lambda_s)^{m_s},$$

there is a unique polynomial $r(t)$ of degree $\le m - 1$ which satisfies the following system of equations:

(2) $$r^{(k)}(\lambda_i) = f^{(k)}(\lambda_i), \quad \begin{array}{l} 1 \le i \le s, \\ 0 \le k \le m_i - 1. \end{array}$$

Moreover, we then have that

$$r(t) = \text{rem}(f, g).$$

Theorem 10 (The Generalized Remainder Formula):
Let $g(t)$ be as in equation (1) above. Then there are unique polynomials

(3) $$e_{ik}(t), \quad \begin{array}{l} 1 \le i \le s, \\ 0 \le k \le m_i - 1, \end{array}$$

of degree $\le m - 1$ such that for any $f \in \mathbb{C}[t]$ we have:

(4) $$\text{rem}(f, g) = \sum_{i=1}^{s} \sum_{k=0}^{m_i - 1} f^{(k)}(\lambda_i) e_{ik}(t).$$