# Evaluating Matrix Polynomials - Method III

This method is based on:

**Theorem 12** (The Spectral Decomposition Theorem): Let $A$ be an $m \times m$ matrix with eigenvalues $\lambda_1, \ldots, \lambda_s$ and multiplicities $m_1, \ldots, m_s$; that is,

$$(1) \qquad \mathrm{ch}_A(t) = (t - \lambda_1)^{m_1} \ldots (t - \lambda_s)^{m_s}.$$

Then there exist $m$ matrices $E_{ik}$ (of size $m \times m$) such that for every polynomial $f(t) \in \mathbb{C}[t]$ we have

$$(2) \qquad f(A) = \sum_{i=1}^{s} \sum_{k=0}^{m_i - 1} f^{(k)}(\lambda_i) E_{ik}.$$

**Remarks:** 1) The matrices $E_{ik}$ depend on $A$ but not on $f$; they are called the constituent matrices of $A$. As we shall see,

$$E_{ik} = e_{ik}(A),$$

where the $e_{ik}(t)$ are the constituent polynomials of $\mathrm{ch}_A(t)$ (cf. Theorem 10).

2) The word "spectral" in the title of the theorem refers to the set *spectrum*$(A) = \{\lambda_1, \ldots, \lambda_s\}$, the set of eigenvalues of $A$. ($\longrightarrow$ Physics)