Comparison of the Three Methods

Method I: based on the Jordan Canonical Form; cf. Theorem 5.

– find P such that $J = P^{-1}AP$ is a $\left\{ \begin{array}{l} \text{diagonal} \\ \text{Jordan} \end{array} \right\}$ matrix; then $f(A) = Pf(J)P^{-1}$. (Use the formula of Theorem 7 to compute $f(J)$.)

– good for diagonable matrices, but usually not practical for non-diagonable matrices.

Method II: based on the Cayley-Hamilton Theorem; cf. Theorem 8.

– find $r(t) = \text{rem}(f, \text{ch}_A)$; then $f(A) = r(A)$. (To compute $r(t)$, use long division (if possible) or the method of substitution or the remainder formula.)

- drawback: one still has to compute $r(A)$ by the naive method.

Method III: based on the Spectral Decomposition Theorem; cf. Theorem 12.

– find the constituent matrices E_{ik} by substituting suitable divisors of $\text{ch}_A(t)$ into the spectral decomposition formula.

– usually the best method.