

# The Cost of Breeding Rabbits

## Situation:

- 1) The females of a certain rabbit species produce 6 females/year.
- 2) These mature in 1 year.
- 3) None die.
- 4) We start with:  $a_0$  adult rabbit,  $b_0$  young.

## Questions:

- 1) How many female rabbits are there after  $n$  years?
- 2) If  $\$c_{k-1}$  is the feeding cost of a mature female in the  $k^{\text{th}}$  year, what is the total cost  $T(n)$  to feed the rabbits for  $n$  years?

## Analysis:

Let  $a_n = \#$  adult females (at the end of) year  $n$   
 $b_n = \#$  females born in year  $n$ .

$$\text{Then: } \underbrace{\begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix}}_{\vec{v}_{n+1}} = \underbrace{\begin{pmatrix} 1 & 1 \\ 6 & 0 \end{pmatrix}}_A \underbrace{\begin{pmatrix} a_n \\ b_n \end{pmatrix}}_{\vec{v}_n}, \quad \text{(recursion relation)}$$

$$T(n) = (1, 0) f(A) \vec{v}_0,$$

$$\text{where } f(t) = c_0 + c_1 t + \dots + c_{n-1} t^{n-1}.$$