Markov Chains as Discrete Linear Systems

Given: 1) A system $\mathcal{S}$ which can be in precisely one of m states $s_1, \ldots, s_m$.

2) A procedure $\mathcal{A}$ which is applied to the system at regular time intervals $t = 0, 1, \ldots$ in such a way that its transition probability (matrix) $A = (a_{ij})$ does not depend on t. Here

a_{ij} = the probability that the procedure takes a system in state s_j to state s_i.

Such a pair $(\mathcal{S}, \mathcal{A})$ is called a Markov chain, and the $m \times m$ matrix A is called the transition matrix of the Markov chain.

Then: If we put $\vec{v}_n = (v_{n,1}, v_{n,2}, \ldots, v_{n,m})^t$, where

$v_{n,i}$ = the probability that the system $\mathcal{S}$ is in state s_i at time $t = n$,

then we have the associated discrete linear system

$$\vec{v}_{n+1} = A\vec{v}_n.$$

Note: The transition matrix $A = (a_{ij})$ is a stochastic matrix: $a_{ij} \geq 0$ and the sum of each column is 1.