Solving Discrete Linear Systems

Theorem 15: The discrete linear system with constant coefficients

(1) $$\vec{u}_{n+1} = A\vec{u}_n + \vec{b}_n, \quad n = 0, 1, \ldots$$

has the (unique) solution

(2) $$\vec{u}_n = A^n\vec{u}_0 + A^{n-1}\vec{b}_0 + A^{n-2}\vec{b}_1 + \ldots + A\vec{b}_{n-2} + \vec{b}_{n-1}.$$

In particular, if the discrete linear system (1) is homogeneous (i.e. $\vec{b}_n = \vec{0}$ for all n), then

(3) $$\vec{u}_n = A^n\vec{u}_0.$$

Method for solving discrete linear systems:

Step 1: Use Theorem 12 to write down the general formula for $\vec{u}_n$ in terms of powers of A.

Step 2: Derive an explicit formula for A^n. (Use the theory of matrix polynomials.)

Note: To solve a difference equation, first transform it into a discrete linear system and then then solve this system by using the above method.