# Equilibrium Points

**Definition:** A vector $\vec{u}_{eq}$ is called an equilibrium point of the discrete linear system

(1) $$\vec{u}_{n+1} = A\vec{u}_n + \vec{b}$$

if it satisfies the equation

(2) $$\vec{u}_{eq} = A\vec{u}_{eq} + \vec{b},$$

or, equivalently, if we have

$$(I - A)\vec{u}_{eq} = \vec{b}.$$

The equilibrium point $\vec{u}_{eq}$ is called stable if we have

(3) $$\vec{u}_n \to \vec{u}_{eq}, \quad \text{as } n \to \infty.$$

**Theorem 16:** Suppose that the d.l.s. (1) has an equilibrium point $\vec{u}_{eq}$. Then, letting

(4) $$\vec{v}_n = \vec{u}_n - \vec{u}_{eq}$$

denote the deviation from the equilibrium point, we have

(5) $$\vec{v}_{n+1} = A\vec{v}_n$$

and hence

(6) $$\vec{u}_n = A^n(\vec{u}_0 - \vec{u}_{eq}) + \vec{u}_{eq}.$$

In particular, if $A^n \to 0$, then $\vec{u}_{eq}$ is stable.