The Jordan Canonical Form

Theorem 4: Every square matrix A is similar to a Jordan matrix. In other words, there is an invertible matrix P such that

$$P^{-1}AP = \mathrm{Diag}(J_{11}, \ldots, J_{ij}, \ldots)$$

is block diagonal matrix consisting of Jordan blocks $J_{ij} = J(\lambda_i, k_{ij})$. Moreover: the J_{ij} are unique up to order, and we have:

1) The $\lambda_1, \ldots, \lambda_s$ are the (distinct) eigenvalues of A.

2) The number of blocks $J_{i1}, J_{i2}, \ldots, J_{ij}, \ldots$ with the same eigenvalue λ_i equals the geometric multiplicity $\nu_i = \nu_A(\lambda_i)$.

3) The sum of the sizes k_{ij} of the blocks $J_{i1}, J_{i2}, \ldots$ equals the algebraic multiplicity $m_i = m_A(\lambda_i)$:

$$k_{i1} + k_{i2} + \ldots + k_{i\nu_i} = m_i.$$

In particular: $\nu_i \leq m_i$.

Corollary: Two $m \times m$ matrices A and B are similar (i.e., $B = P^{-1}AP$, for some P) if and only if they have the same Jordan canonical form (up to order).