Power Convergent Matrices (Special Cases)

Definition: A square matrix A is called power convergent if $\lim_{k\to\infty} A^k$ exists.

Theorem 4: A diagonal matrix $A = \mathrm{Diag}(\lambda_1, \ldots, \lambda_m)$ is power convergent if and only if for each i with $1 \le i \le m$ we have:

$$\text{either} \quad |\lambda_i| < 1 \quad \text{or} \quad \lambda_i = 1. \tag{1}$$

Corollary. A diagonable matrix A is power convergent if and only if each eigenvalue λ_i satisfies (1).

Theorem 5: A Jordan block $J = J(\lambda, m)$ is power convergent if and only if

$$\text{either} \quad |\lambda| < 1 \quad \text{or} \quad \lambda = 1 \text{ and } m = 1. \tag{2}$$

Remark. The proof of Theorem 5 uses the fact that

$$|\lambda| < 1 \quad \Rightarrow \quad \lim_{n\to\infty} \binom{n}{a} \lambda^{n-a} = 0, \text{ for all } a \in \mathbb{N}.$$

Corollary. If $|\lambda| < 1$, then $\lim_{k\to\infty} J(\lambda, m)^k = 0$.