Simple Eigenvalues

Definition: An eigenvalue λ of A is called simple if its algebraic multiplicity $m_A(\lambda) = 1$.

Remark. Clearly, each simple eigenvalue is regular.

Theorem 10: If A is power convergent and 1 is a simple eigenvalue of A, then

$$\lim_{n\to\infty} A^n = E_{10} = \frac{1}{\underbrace{\vec{u}^t\vec{v}}_{\text{scalar}}}\underbrace{\vec{u}\vec{v}^t}_{\text{matrix}},$$

where:

$\vec{u} \in E_A(1)$ is any non-zero 1-eigenvector of A,
$\vec{v} \in E_{A^t}(1)$ is any non-zero 1-eigenvector of A^t.

Remark. Since $\det(C^t) = \det(C)$ for any matrix C, it follows that

$$\mathrm{ch}_{A^t}(t) = \mathrm{ch}_A(t).$$

Thus: A and A^t have the same eigenvalues with the same algebraic multiplicities. (In fact, A and A^t are similar, so they even have the same Jordan canonical form.)