Primitive Stochastic Matrices

Definition: A square stochastic matrix A is called primitive if for some $n \geq 1$ the matrix A^n has no entries equal to 0.

Perron's Theorem: (O. Perron, 1907) If A is a primitive stochastic matrix, then $\lambda_1 = 1$ is a simple, dominant eigenvalue of A and the associated eigenspace is generated by a unique stochastic vector $\vec{p}$ called the Perron vector of A; i.e.

$$E_A(1) = \{c\vec{p}\}.$$

Furthermore, $\vec{p}$ has the property that none of its entries is equal to 0.

Corollary: If A is primitive and stochastic, then A is power convergent with limit

$$\lim_{n\to\infty} A^n = \underbrace{(\vec{p}|\vec{p}|\ldots|\vec{p})}_{m},$$

where $\vec{p}$ denotes the Perron vector of A. Moreover, for any vector $\vec{v}$ we have

$$\lim_{n\to\infty} A^n\vec{v} = c\vec{p}, \quad \text{where } c = \Sigma_m\vec{v}.$$