Thereom 5.11 (Lagrange-Sylvester Interpolation Formula) Suppose that we are given

$$\begin{aligned} \lambda_1, \ldots, \lambda_s &\in \mathbb{C} \\ m_1, \ldots, m_s &\in \mathbb{N} \\ c_{ik} &\in \mathbb{C}, \quad 1 \le i \le s,\ 0 \le k \le m_i - 1, \end{aligned}$$

where the λ_i's are distinct. Then there is a unique polynomial $h(t) \in \mathbb{C}[t]$ of degree $\le m-1$, where $m = m_1 + \cdots + m_s$, such that

$$(1) \qquad h^{(k)}(\lambda_i) = c_{ik}, \quad \text{for } 1 \le i \le s,\ 0 \le k \le m_i - 1.$$

Theorem 5.11, as stated on page 241 of the text goes on to give an explicit formula for $h(t)$, namely $h(t)$ is given by the formula

$$h(t) = \sum_{i=1}^{s} \sum_{k=0}^{m_i-1} c_{ik} e_{ik}(t),$$

where the $e_{ik}(t)$ are the constituent polynomials of $g(t) = (t-\lambda_i)^{m_i} \cdots (t-\lambda_s)^{m_s}$.

The constituent polynomials are defined in Theorem 5.10, which we have not proved. Instead, we will give a direct proof of Theorem 5.11 (at least, in a special case from which the general proof will hopefully be clear). Then we will state what the constituent polynomials are in terms of Theorem 5.11.