Math 211

Assignment 2

Due 1 October 2021

[5] 1. Use the Euclidean algorithm to find:

(a) the greatest common divisor of 7919 and 7927;

(b) integers x and $y \in \mathbb{Z}$ such that $7919x + 7927y = 1$;

(c) integers x and $y \in \mathbb{Z}$ such that $7919x + 7927y = 10$.

[5] 2. Use the GCD-criterion to decide which of the following equations has an integral solution $x, y \in \mathbb{Z}$. Furthermore, find such a solution for each equation which is solvable.

(a) $294x + 816y = -9$;

(b) $777x - 1628y = 37$;

(c) $1233x + 2115y = 18$.

[6] 3. Is each of the following linear Diophantine equations solvable in integers $x, y \in \mathbb{Z}$? If so, write down the general solution. If not, explain why not.

(a) $8023x - 8249y = 1243$;

(b) $1079x + 1411y = 243$;

(c) $123456x - 654321y = 7$.

[1] 4. Verify that $(x, y, z) = (2m + 1, 2m(m + 1), 2m(m + 1) + 1)$ is a Pythagorean triple for every $m \geq 1$. What are the triples corresponding to the first four values of m?

[3] 5. MAPLE problem (refer to the MAPLE instruction sheet):

Consider the following MAPLE commands:

```
euclid:= proc(m,n) local q,r1,r2,r3;
    r1:=m; r2:=n;
    while r2 <> 0 do;
        q:=iquo(r1,r2); r3:= irem(r1,r2);
        lprint(r1, `=`, q, `*`, r2, `+`, r3);
        r1:=r2; r2:=r3; od;
    RETURN(r1); end;
```

Implement this program on the computer and explain what the program does. (Use the " text insertion" feature of MAPLE.) Also, test the program on your favourite pair of 4-digit numbers (your choice). In addition, test it for $m = 1122334455667788997531$ and $n = 97531246801234567 89$.

Note: The symbol ‘ in the above commands is a "backwards quote" which is found on the left hand side of your keyboard. Do not confuse it with the "forward quote" ’ which is on the right hand side of your keyboard.