Math 211

Assignment 3

Reminder: Mid-term test on Wednesday, 20 October 2021 at 1:30 p.m. in Walter Light Hall, Room 210.

[3] 1. A movie theatre charges $3.75 for children and $9.00 for adults. Last Saturday, the manager noticed that that he took in exactly $165 for the afternoon performance and that there had been more adults than children in the theatre. What was the exact number of children in attendance that afternoon?

[4] 2. A cashier, in paying out a cheque, inadvertently interchanges the dollar and cents amounts, thus giving the recipient in dollars the amount shown on the cheque in cents, and in cents the amount marked in dollars. On the way home the recipient spent $3.50. Counting his money at home, he found to his surprise that he now had twice the amount the cheque had called for. What was the original amount of the cheque?
   [Hint: Show that this leads to the linear Diophantine equation $98x - 199y = 350$.]

[4] 3. Find the general integer solution of each of the following Diophantine equations:
   (a) $4x + 6y - 9z = 20$;   (b) $3x + 9y + 12z = 20$.

[4] 4. When Mr. Smith returned from a trip in Europe in 1966, he found that he had in his possession 35 British sixpence coins, 55 French ten-centimes pieces and 77 Greek drachmas. Mr. Smith converted each of theses coins to its value in Canadian money (rounded off to the nearest cent) and found that the total was worth $5.86. How much was each coin worth in 1966 (to the nearest cent)?

[2] 5. Use the sieve of Eratosthenes to find all the prime numbers less than 120, and list all the twin prime pairs in that range.

[1] 6. Find the prime factorization of the following integers by hand (show your work):
   (a) 1728;   (b) 684600.

[2] 7. Prove the following formula by induction on n:

$$1^2 + 2^2 + \ldots + n^2 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n.$$