

Math 211

Assignment 4

Due 29 October 2021

- [2] 1. Prove that $\sqrt[3]{40}$ is irrational.
- [3] 2. Let $m = 2^5 3^7 11^3 17^2$ and $n = 3^2 5^3 7^4 13^2$.
- (a) Find $\gcd(m, n)$.
 - (b) Find the list of all divisors of n .
 - (c) Write down the prime decomposition of mn .
- [5] 3. Find the following remainders (by hand). Explain your method by writing down your intermediate calculations.
- (a) $\text{rem}(2^{12}, 63)$.
 - (b) $\text{rem}(8^5 + 9^5, 17)$.
 - (c) $\text{rem}(3^8 - 4^8, 10)$.
 - (d) $\text{rem}(24 \cdot 25 + 27 \cdot 29, 26)$.
 - (e) $\text{rem}(103 \cdot 65 - 329 \cdot 663, 33)$.

- [2] 4. Use the casting-out nines method to show that

$$234785346 \cdot 5683592187 \neq 1334424157147691702.$$

(Explain and justify your method.)

- [4] 5. Use the power-mod algorithm to find the following remainders by hand.

$$(a) \text{rem}(5^{18}, 11); \quad (b) \text{rem}(7^{14}, 18).$$

- [4] 6. (a) Use the MAPLE commands `ifactor(.)` and `ifactors(.)` to find the prime factorization of the number 123456789. Explain your output and comment on the similarity and difference between the two results obtained.
- (b) Use the commands `ithprime(.)` and `[seq(.)]` to write a program `firstprimes(n)` which returns the list of the first n primes. Use it to find the first 20 and also the first 50 primes.