Math 211

Assignment 7

Due 19 November 2021

[3] 1. Express in rectangular form:

(a) $(2+3i)(3-2i)$; (b) $\dfrac{(1+i)(2-3i)}{3+i}$; (c) $\dfrac{2i(-8+6i)^2}{(4-2\sqrt{5}i)(2-4\sqrt{5}i)}$.

[3] 2. (a) Express $\alpha = 1+i$ and $\beta = 1+i\sqrt{3}$ in polar form.

(b) Find the polar form of $\alpha\beta$ and β/α, where α and β are as in part (a).

(c) Find $\arg(1-i)$ and $\arg(-1+\sqrt{3}i)$.

[3] 3. (a) Use properties of complex conjugation to verify the identity

$$(x^2+y^2)(a^2+b^2) = (xa-yb)^2 + (xb+ya)^2$$

for real numbers $x, y, a, b \in \mathbb{R}$. [Hint: consider $z_1 = x+iy, z_2 = a+ib$.]

(b) Show that if n and $m \in \mathbb{Z}$ are two integers which are sums of two squares (of integers), then so is their product mn.

[10] 4. (a) Write a MAPLE program `encode`(m, n, e) which uses the RSA method with public key (n, e) to encode a given message m. (Here, m is an integer with $0 < m < n$.)

(b) Recall from class that the message *this is top secret* was translated as the number sequence $[20080919, 00091900, 20151600, 19050318, 05200000]$. Use your program of part (a) to encode this message (or number sequence) with the public key $n = 2364389329$ and $e = 123456787$.

(c) Recall that if n can be factored, then the secret code d can be computed. Thus, using the MAPLE command `ifactors`(n), write a short program `crack`(n, e) which determines the secret code d needed for decoding the messages encoded in (a).

(d) Using your programs, find d for the example of (b), and use this to decode the message encoded above. (Check that you obtain the original message again!)

(e) Using your programs, decode (and interpret) the following secret message which Bob sent to Alice, whose public key is $(n_A, e_A) = (50012305249537, 22331122334567)$:

$$M1 = 45435724602850 \quad M2 = 32563574741091$$
$$M3 = 46308572459732 \quad M4 = 14201357670970$$

Note: You can interpret the decoded message either "by hand" or (for MAPLE experts) by using the programs given in the text. In either case, use MAPLE's text feature to write your conclusions in your print-out.