Math 211

Assignment 8

Due 26 November 2021

[2] 1. Let $f(z) = z^4 + 4$. Verify that $f(1+i) = 0$, and conclude from this by using a result from class (without any further computations) that also $f(1-i) = 0$. Moreover, find two other solutions of the equation $f(z) = 0$.

[4] 2. Find and sketch all the complex solutions of the equation $z^4 = i$.

[4] 3. Find all the complex solutions of each of the following equations:

(a) $x^3 = -i$;

(b) $x^3 + 3x^2 + 3x + 1 + i = 0$.

[Hint for part (b): use (a) and the fact that $(x+1)^3 = x^3 + 3x^2 + 3x + 1$.]

[6] 4. Find the quotient and the remainder when you divide:

(a)	$x^9 + x^7 + 2x^5 + 2x^3 + x + 1$	by	$x^4 + x^2 + 1$	in	$\mathbb{Q}[x]$,
(b)	$x^5 + x^3 - x^2 + x$	by	$x^2 + i$	in	$\mathbb{C}[x]$,
(c)	$x^3 - x^2 - x$	by	$2x^2 + 1$	in	$\mathbb{F}_5[x]$.

[2] 5. Let $f, g \in F[x]$ be two polynomials with $g \neq 0$ and let $c \in F$, $c \neq 0$. Prove that

$$\mathrm{quot}(f, cg) = \frac{1}{c}\mathrm{quot}(f, g) \quad \text{and} \quad \mathrm{rem}(f, cg) = \mathrm{rem}(f, g).$$

[2] 6. Let $f, g \in R[x]$ (where $R = \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ or $\mathbb{F}_p$) be monic polynomials of the same degree. Prove that $f|_R\, g$ if and only if $f = g$.