Math 211

Assignment 9

Due 3 December 2021

[5] 1. Find the remainder when $x^{1999} + x$ is divided by:

(a) $2x + 1$; (b) $x^2 - 3x + 2$; (c) $x + \frac{-1+i\sqrt{3}}{2}$; (d) $x^2 + x + 1$.

[3] 2. Prove that the following polynomials are irreducible over the given field:

(i) $x^2 - x + 2 \in \mathbb{Q}[x]$; (ii) $x^2 - 2 \in \mathbb{F}_5[x]$; (iii) $x^3 - 2 \in \mathbb{F}_7[x]$.

[2] 3. Let $f(x) = (x-1)^3(x^2+x-2)^2(x^2+x+1)^3$ and $g(x) = (x+2)^2(x^2-3x+2)^3(x^2+x+1)^4$.

(a) What are the roots of f in $\mathbb{C}$, and the multiplicities of each?

(b) Find the gcd of f and g without using the Euclidean algorithm.

[4] 4. Express $f(x) = x^3 + 3x^2 + 2x + 6$ as a product of irreducible factors:

(a) in $\mathbb{Q}[x]$; (b) in $\mathbb{R}[x]$; (c) in $\mathbb{C}[x]$; (d) in $\mathbb{F}_3[x]$.

In each case, justify that the factors which you have found are indeed irreducible.

[3] 5. (a) Express $f(x) = x^5 - 1$ as a product of irreducible factors in $\mathbb{C}[x]$. (You can leave the coefficients of your answer in terms of sines and cosines, if you want.)

(b) Use your answer in (a) to find the factorization of $f(x)$ in $\mathbb{R}[x]$.

[3] 6. Given that $\alpha = -1 + i\sqrt{2}$ is a root of the polynomial

$$f(x) = x^5 + 3x^4 + 6x^3 + 2x^2 - 3x - 9,$$

factor the polynomial into a product of monic irreducible polynomials:

(a) in $\mathbb{Q}[x]$; (b) in $\mathbb{R}[x]$; (c) in $\mathbb{C}[x]$.