Math 211

Assignment 7 - Solutions

[3] 1. (a) $(2+3i)(3-2i) = 2(3-2i) + 3i(3-2i) = 6 - 4i + 9i - 6i^2 = 6 + 5i - 6(-1) = 12 + 5i.$
[Alternately: by the formula we have $(2+3i)(3-2i) = (2(3) - 3(-2)) + i(2(-2) + 3(3)) = 12 + 5i.$]

(b), (c) By multiplying top and bottom by the complex conjugate of the denominator, we get:

(b) $\frac{(1+i)(2-3i)}{3+i} = \frac{(1+i)(2-3i)(3-i)}{3^2+1^2} = \frac{(5-i)(3-i)}{10} = \frac{7}{5} - \frac{4}{5}i.$

(c) $\frac{2i(-8+6i)^2}{(4-2\sqrt{5}i)(2-4\sqrt{5}i)} = \frac{2i(-8+6i)^2(4+2\sqrt{5}i)(2+4\sqrt{5}i)}{(16+4\cdot 5)(4+16\cdot 5)} = \frac{2i(28-96i)(-32+i(20\sqrt{5}))}{36\cdot(7\cdot 12)}$
$= \frac{2}{27\cdot 7}\left[i(7-24i)(-8+i5\sqrt{5}\right] = \frac{2}{27\cdot 7}\left[(-24\cdot 8 - 7\cdot 5\sqrt{5}) + i(24\cdot 5\sqrt{5} - 8\cdot 7)\right]$
$= -\left(\frac{128}{63} + \frac{10}{27}\sqrt{5}\right) + i\left(\frac{80}{63}\sqrt{5} - \frac{16}{27}\right)$ (Alternately: $= -\left(\frac{384+70\sqrt{5}}{189}\right) + \frac{-112+240\sqrt{5}}{189}i.$)

[3] 2. (a) $\alpha = \sqrt{2}(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4}))$, $\beta = 2(\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3}))$.

(b) Using part (a) and the multiplication formula we obtain that $\alpha\beta = 2\sqrt{2}(\cos(\frac{\pi}{4}+\frac{\pi}{3}) + i\sin(\frac{\pi}{4}+\frac{\pi}{3})) = 2\sqrt{2}(\cos(\frac{7\pi}{12}) + i\sin(\frac{7\pi}{12}))$. Similarly, $\beta/\alpha = (2/\sqrt{2})(\cos(\frac{\pi}{3}-\frac{\pi}{4}) + i\sin(\frac{\pi}{3}-\frac{\pi}{4})) = \sqrt{2}(\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12}))$.

(c) Since $1-i$ is in the fourth quadrant, we obtain $\arg(1-i) = 2\pi + \arctan(\frac{-1}{1}) = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$. Similarly, since $-1+i\sqrt{3}$ lies in the second quadrant, we have $\arg(-1+i\sqrt{3}) = \arctan(\frac{\sqrt{3}}{-1}) + \pi = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$.

Note: Do not use approximations in your calculations unless this is necessary.

[2] 3. (a) Put $z_1 = x + iy, z_2 = a + bi$. Then

$$(x^2+y^2)(a^2+b^2) = |z_1|^2|z_2|^2 = z_1\bar{z}_1 z_2\bar{z}_2 = (z_1z_2)\overline{z_1z_2} = |z_1z_2|^2 = \mathrm{Re}(z_1z_2)^2 + \mathrm{Im}(z_1z_2)^2.$$

From the formula in class we know that $\mathrm{Re}(z_1z_2) = xa - yb$ and $\mathrm{Im}(z_1z_2) = xb + ya$, so the identity follows.

[1] (b) By hypothesis, $n = x^2 + y^2, m = a^2 + b^2$ with $x, y, a, b \in \mathbb{Z}$. From the formula in (a) we obtain that $nm = (xa - yb)^2 + (xb + ya)^2$ is the sum of two integer squares since the sum and product of integers is again an integer.

[10] 4. See the MAPLE solution on the course Web site (www.mast.queensu.ca/~math211).