1. Assuming that $p$ and $r$ are false and that $q$ and $s$ are true, find the truth value of each of the following statement forms.

(a) $\neg(p \rightarrow q)$.
(b) $(p \rightarrow q) \land (q \rightarrow r)$.
(c) $(p \rightarrow q) \rightarrow r$.
(d) $(s \rightarrow (p \land \neg r)) \land ((p \rightarrow (r \lor q)) \land s)$.

2. Reduce the following sentences to statement forms.

(a) A necessary condition for $x$ to be prime is that $x$ is odd or $x = 2$.
(b) A sufficient condition for $f$ to be continuous is that $f$ is differentiable.
(c) Grass will grow only if enough moisture is available.
(d) If taxes are increased or government spending decreases, then inflation will not occur this year.

3. Determine which of the following are tautologies, which are contradictions, and which are neither.

(a) $[(a \rightarrow b) \rightarrow \neg(b \rightarrow a)] \leftrightarrow (a \leftrightarrow b)$.
(b) $[(a \rightarrow b) \rightarrow b] \rightarrow b$.
(c) $b \land \neg(a \lor b)$.
(d) $(a \rightarrow (b \land \neg b)) \rightarrow \neg a.$
4. Using basic logical equivalences (identities), show the following.

(a) $\neg (p \leftrightarrow q) \iff (p \land \neg q) \lor (\neg p \land q)$.

(b) $\neg [p \rightarrow (q \rightarrow r)] \iff p \land q \land \neg r$.

(c) $\neg [(p \land (q \rightarrow r)) \lor (\neg q \land p)] \iff \neg p \lor (q \land \neg r)$.

5. Determine whether the following argument is valid by representing the sentences as statement forms and checking to see whether the conjunction of the assumptions logically implies the conclusion.

For the Prime Minister to be reelected it is sufficient that he provides substantial tax cuts. He will provide substantial tax cuts only if the budget surplus is larger than $10$ billion. But the budget surplus is smaller than $10$ billion. Therefore, the Prime Minister will not be reelected.

Recommended Practice Problems:

1. Page 136, # 1. (Humphreys-Prest, 2nd Ed.)

2. Page 137, # 2. (Humphreys-Prest, 2nd Ed.)

3. Page 137, # 3. (Humphreys-Prest, 2nd Ed.)

4. Show the following using the basic logical identities in Theorem 1.22 of the Lecture Notes.

(a) $\neg p \leftrightarrow q \iff (\neg q) \leftrightarrow p$;
(b) $((p \rightarrow \neg q) \land (p \rightarrow \neg r)) \iff \neg (p \land (q \lor r))$;
(c) $p \rightarrow (q \lor r) \iff (\neg q) \rightarrow (\neg p \lor r)$. 

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