MTHE 217 - Algebraic Structures with Applications  
Fall 2019

Homework # 2

**Material:** *Quantifiers and Set theory*

**Readings:** Section 2 (Lecture Notes) and Sections 3.2, 3.3 and 2.1 (Humphreys-Prest text)

1. List the elements of the following sets: $A = \{x \in \mathbb{Q} : x^2 = 2\}$ and $B = \{n \in \mathbb{N} : 2 < 3n + 1 < 20\}$, where $\mathbb{Q}$ denotes the set of rational numbers and $\mathbb{N}$ denotes the set of natural numbers.

2. Let $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3\}$, and $C = \{2, 4\}$. Find all sets $X$ satisfying each pair of conditions.
   
   (a) $X \subseteq A$ and $X \nsubseteq B$.
   
   (b) $X \subset B$ and $X \nsubseteq C$.

3. Let $A$, $B$, $C$ and $D$ be sets in a universe $U$. Prove:
   
   (a) $(A \cap B)^c = A^c \cup B^c$ (de Morgan’s law).
   
   (b) $(A \times C) \cap (B \times D) = (A \cap B) \times (C \cap D)$.
   
   (c) If $A \cap C = B \cap C$ and $A \cup C = B \cup C$, then $A = B$.
   
   (d) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

4. Show that for any sets $A$ and $B$ in a universe $U$,

   $$A \setminus (A \setminus B) = A \cap B$$

   by using elementary properties of set operations (such as de Morgan’s law, distributivity, etc).
5. Let \( A = \{x, y\} \), \( B = \{0, 1\} \) and \( C = \{-1, 0, 1\} \). Find these sets:

(a) \( A \times B \).

(b) \( A \times B \times C \).

(c) \( P(B) \times P(B) \), where \( P(B) \) is the power set of \( B \).

(d) \( P(A \times B) \).

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**Recommended Practice Problems:** (Do not hand in)

1. Page 86, # 1. (Humphreys-Prest, 2nd Ed.)
2. Page 86, # 2. (Humphreys-Prest, 2nd Ed.)
3. Page 86, # 3. (Humphreys-Prest, 2nd Ed.)
4. Page 86, # 4. (Humphreys-Prest, 2nd Ed.)
5. Page 86, # 6. (Humphreys-Prest, 2nd Ed.)
6. Page 86, # 9. (Humphreys-Prest, 2nd Ed.)