1. Let \( p \) be a prime number. Show the following:

   (a) If \( p > 2 \), then either \( p = 4n + 1 \) or \( p = 4n + 3 \) for some \( n \in \mathbb{N} \).
   
   (b) If \( p > 3 \), then either \( p = 6n + 1 \) or \( p = 6n + 5 \) for some \( n \in \mathbb{N} \).
   
   (c) If \( p > 2 \), then \( p^2 = 4n + 1 \) for some \( n \in \mathbb{N} \).

2. In each case, compute \( \gcd(m, n) \) and express it as a linear combination of \( m \) and \( n \).

   (a) \( m = 377 \) and \( n = 29 \).
   
   (b) \( m = -231 \) and \( n = 150 \).

3. If \( m \) and \( n \) are odd integers, show that \( m^2 - n^2 \) is divisible by 8.

4. (a) If \( d > 0 \), \( d | (11k + 4) \) and \( d | (10k + 3) \) for some integer \( k \), show that \( d = 1 \) or \( d = 7 \).
   
   (b) If \( d > 0 \), \( d | (35k + 26) \) and \( d | (7k + 3) \) for some integer \( k \), show that \( d = 1 \) or \( d = 11 \).

5. Let \( m \) and \( n \) be integers, and write \( d = \gcd(m, n) \).

   (a) Verify that \( \frac{m}{d} \) and \( \frac{n}{d} \) are integers, and show that they are relatively prime.
   
   (b) If \( k | d \) and \( k > 0 \), show that \( \gcd\left(\frac{m}{k}, \frac{n}{k}\right) = \frac{d}{k} \).

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**Recommended Practice Problems:** (Do not hand in)

1. Page 15, \# 1. (Humphreys-Prest, 2nd Ed.)

2. Page 15, \# 3. (Humphreys-Prest, 2nd Ed.)
5. Suppose that $p \geq 2$ is an integer with the following property: if $m$ and $n$ are integers with $p | mn$, either $p | m$ or $p | n$. Show that $p$ must be a prime. (Hint: use a proof by contradiction.)