STUDENT NUMBER:

Math/Mthe 225 — Quiz 1

30 September 2019

Professor E. Kani

- This is a 35 minute test.
- No textbooks, notes, or aids other than your calculator are allowed.
- Do not turn the first page until instructed by your proctor.
- For full marks, you must show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise.
- Wherever appropriate, include units in your answers.
- While waiting to start, **please fill in your name and student number above.**
- If you need more room, there is a blank page at the end of the test. **If you use these pages, you must provide clear directions to the marker**, e.g. "Continued on page 4".

Q1	Q2	Total
5	5	10

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Question 1.

Verify by substitution that the function $y(t) = Ce^{-t^3}$ is a solution of the differential equation

$$y' + 3t^2 y = 0.$$

Moreover, determine the constant C such that $y(t)$ satisfies the initial value problem

$$y' + 3t^2 y = 0; \quad y(0) = 10.$$

Solution: We have that $y'(t) = \frac{d}{dt}(Ce^{-t^3}) = (-3t^2)(Ce^{-t^3}) = -3Ct^2e^{-t^3}$, so

$$y'(t) + 3t^2 y = (-3t^2 Ce^{-t^3}) + 3t^2(Ce^{-t^3}) = (-3t^2 + 3t^2)Ce^{t^3} = 0.$$

Substituting the condition $y(0) = 10$ for $y(t)$ yields $10 = y(0) = Ce^{-0^3} = C$, so $C = 10$.

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Question 2.

A 5 lb roast initially at 50F is placed in an oven that is at 375F. After 75 minutes it is found that the temperature of the roast is 125F. What will the temperature of the roast be 100 minutes after it was put in the oven?

Solution. Let $T(t)$ be the temperature of the roast at time t. Then the differential equation that describes this situation is

$$\frac{dT}{dt} = k(A - T),$$

where k is the constant of proportionality and A is the ambient temperature.
We are given that $A = 375$, $T(0) = 50$, $T(75) = 125$, and need to find $T(100)$. For this, we first solve the separable differential equation:

$$\frac{dT}{dt} \;=\; k(375 - T).$$

Dividing both sides by $375 - T$ and integrating gives $\int \frac{dT}{375-T} = \int k dt$. Thus

$$-\ln|375 - T| \;=\; kt + \ln C \quad \text{or} \quad 375 - T \;=\; Ce^{-kt}.$$

Thus, the general solution is $T(t) = 375 - Ce^{kt}$. Now $T(0) = 50$ implies that $375 - 50 = C$, so $C = 325$. Moreover, since $T(75) = 125$ we see that $250 = 325e^{-75k}$. Solving for k, we get

$$k = \frac{\ln(250/325)}{-75} \approx 0.0035.$$

Therefore, the solution is

$$T(t) = 375 - 325e^{-0.0035t}.$$

Finally, we calculate the temperature of the roast after 100 minutes by plugging $t = 100$ in the above equation:

$$T(100) = 375 - 325e^{-0.35} \approx 146.$$

So, the roast would be around 146F after 100 minutes.

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Space for additional work. **Indicate clearly which question you are continuing if you use this space.**