Ex. 6: Using Laplace transforms, solve the IVP

$$y'' - y' - 2y = 0, \quad y(0) = 1, \; y'(0) = 0.$$

Sol'n: Put $Y(s) = \mathcal{L}\{y(t)\}$. Then by (1), (2):

$$\mathcal{L}\{y'\} \overset{(1)}{=} s\mathcal{L}\{y\} - y(0) = sY(s) - 1$$

$$\mathcal{L}\{y''\} \overset{(2)}{=} s^2\mathcal{L}\{y\} - s\,y(0) - \underline{y'(0)} = s^2 Y(s) - s \cdot 1$$

Thus, applying $\mathcal{L}\{\cdot\cdot\}$ to both sides of the DE:

$$[s^2 Y(s) - s] - [sY(s) - 1] - 2Y(s) = 0$$

or $$(s^2 - s - 2)Y(s) = s - 1,$$

or $$Y(s) = \frac{s-1}{s^2 - s - 2}.$$

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Thus:

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{s-1}{s^2-s-2}\right\}.$$

To compute the latter, note that

$$s^2-s-2 = (s+1)(s-2).$$

Thus (by the partial fraction decomposition method) there exist A, B s.th.

$$\frac{s-1}{(s+1)(s-2)} = \frac{A}{(s+1)} + \frac{B}{(s-2)} = \frac{A(s-2)+B(s+1)}{(s-1)(s-2)}$$

so

$$s-1 = A(s-2)+B(s+1) = (A+B)s + B-2A$$

We thus have (by comparing coefficients)

$$A+B=1, \quad B-2A=-1$$

$\Rightarrow$ $3A=2$, $A = 2/3$ and $B = 1/3$. Thus

$$\frac{s-1}{s^2-s-2} = \frac{2}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s-2}$$

so $\mathcal{L}^{-1}\{\ldots\} = \frac{2}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + 1/3\,\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$

$$= \frac{2}{3}e^{-t} + 1/3\, e^{2t}.$$

Thus, the sol'n of the IVP is

$$y(t) = 2/3\, e^{-t} + 1/3\, e^{2t}$$