MATH/MTHE 225: Formula Sheet

Elementary Differential and Integral Calculus

Exponents

$x^a \cdot x^b = x^{a+b}, \quad x^a \cdot y^a = (xy)^a, \quad (x^a)^b = x^{ab}, \quad x^0 = 1.$

Logarithms

$\ln(xy) = \ln(x) + \ln(y), \quad \ln(x^a) = a\ln(x), \quad \ln(1) = 0, \quad e^{\ln(x)} = x, \quad \ln(e^y) = y, \quad a^x = e^{x\ln(a)}.$

Trigonometry

$\cos(0) = \sin\left(\frac{\pi}{2}\right) = 1, \quad \sin(0) = \cos\left(\frac{\pi}{2}\right) = 0.$

$\cos(-\theta) = \cos(\theta), \quad \sin(-\theta) = -\sin(\theta), \quad \cos^2(\theta) + \sin^2(\theta) = 1.$

$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi), \quad \cos(2\theta) = \cos^2(\theta) - \sin^(\theta).$

$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi), \quad \sin(2\theta) = 2\sin(\theta)\cos(\theta).$

$\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}, \quad \sec(\theta) = \dfrac{1}{\cos(\theta)}, \quad 1 + \tan^2(\theta) = \sec^2(\theta).$

Inverse Trigonometric Functions

$y = \arcsin(x)$ means $x = \sin(y)$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

$y = \arccos(x)$ means $x = \cos(y)$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

$y = \arctan(x)$ means $x = \tan(y)$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Note. The function $\arcsin(x)$ is often denoted by $\sin^{-1}(x)$, but this notation can lead to confusion because $\sin^{-1}(x) \neq (\sin(x))^{-1}$, whereas $\sin^2(x) = (\sin(x))^2$. Similarly, $\arccos(x)$ is denoted by $\cos^{-1}(x)$ and $\arctan(x)$ is denoted by $\tan^{-1}(x)$.

Lines in $\mathbb{R}^2$

The line $y = mx + c$ has slope m and passes through the point $(0, c)$.

The line through (x_1, y_1) with slope m has equation $y - y_1 = m(x - x_1)$.

The line through (x_1, y_1) and (x_2, y_2) has slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and equation $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

The Binomial Formula

$(x + y)^2 = x^2 + 2xy + y^2, \quad (x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$, and so on.

The coefficients in $(x + y)^n$ form the n-th row of *Pascal's triangle*:

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1 \\
\end{array}
$$

. . . .

The Quadratic Formula

If $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Calculus

If $u = u(x)$ and $v = v(x)$ are functions of x and $u' = \frac{du}{dx}$, $v' = \frac{dv}{dx}$, then we have

Product formula: $(uv)' = uv' + u'v$.

Quotient formula: $\left(\frac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$.

Integration by parts formula: $\int uv'dx = uv - \int u'vdx$.

If $y = y(u)$ is a function of u, and $u = u(x)$ is a function of x, then the *chain rule* states that

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \quad \text{and hence} \quad \int y\frac{du}{dx}dx = \int ydu.$$

Standard Derivatives and Integrals

If $y = x^a$, then $y' = ax^{a-1}$, and $\int x^a dx = \dfrac{x^a}{a+1} + C$, provided that $a \neq -1$.

If $y = e^x$, then $y' = e^x$, and $\int e^x dx = e^x + C$.

If $y = \ln(x)$, then $y' = \dfrac{1}{x}$, and $\int \dfrac{1}{x}dx = \ln(|x|) + C$.

If $y = \sin(x)$, then $y' = \cos(x)$, and $\int \sin(x)dx = -\cos(x) + C$.

If $y = \cos(x)$, then $y' = -\sin(x)$, and $\int \cos(x)dx = \sin(x) + C$.

If $y = \tan(x)$, then $y' = \sec^2(x)$, and $\int \tan(x)dx = \ln(|\sec(x)|) + C$.

If $y = \arcsin(x)$, then $y' = \dfrac{1}{\sqrt{1-x^2}}$, and $\int \dfrac{1}{\sqrt{1-x^2}}dx = \arcsin(x) + C$.

If $y = \arccos(x)$, then $y' = \dfrac{-1}{\sqrt{1-x^2}}$.

If $y = \arctan(x)$, then $y' = \dfrac{1}{1+x^2}$, and $\int \dfrac{1}{1+x^2}dx = \arctan(x) + C$.