

Problem 1: (text, Problem 3, p. 181)

- see p. 106 of notes

Problem 2: A mass of 3 kg is attached to a spring with $k = 63 \text{ N/m}$ and a damper with 30 N per m/s of velocity. The mass is set in motion at $x_0 = 2 \text{ m}$ with $v_0 = 2 \text{ m/s}$. Find $x(t)$ and the nature of damping.

Sol'n: Here we have the DE

$$3x'' + 30x' + 63x = 0, \quad x(0) = x'(0) = 2.$$

The char. poly. is

$$3X^2 + 30X + 63 = 3(X^2 + 10X + 21)$$

$$= 3(X+3)(X+7)$$

so $r_1 = -3$ and $r_2 = -7$ are the roots. $\Rightarrow$ overdamped. Thus

$$x(t) = c_1 e^{-3t} + c_2 e^{-7t}$$

$$2 = x(0) = c_1 + c_2$$

$$2 = x'(0) = c_1(-3) + c_2(-7)$$

$$\left. \begin{array}{l} c_1 + c_2 = 2 \\ -3c_1 - 7c_2 = 2 \end{array} \right\}$$

Mult. 1st eq'n by 3 and adding $\Rightarrow -4c_2 = 8$

$$\Rightarrow c_2 = -2, c_1 = 4 \quad \text{so} \quad x(t) = 4e^{-3t} - 2e^{-7t}.$$