MATH/MTHE 225: Course Summary

Week 1: Section 1.1: Definition and examples of differential equations (DEs). Terminology: Ordinary DEs, order of a DE, solution of a DE, linear DEs, initial value problems (IVPs), initial conditions. Verifying that a given solution solves a given DE. Mathematical models.

Section 1:2: Solving a DE of the form $\frac{dy}{dx} = f(x)$ by integration.

Section 1.4: Solving separable DEs: $\frac{dy}{dx} = f(x)h(y)$. Implicit solutions of DEs.

Week 2: Section 1.4 (cont'd): Applications of separable DEs: Newton's Law of Heating/Cooling. Radioactive decay, population growth.

Section 1.5: Linear first order DEs. Solving a such a DE by finding an integrating factor. The Existence/Uniqueness Theorem for a linear first order IVP.

Week 3: Section 1.5 (cont'd): Application: Mixture Problems.

Section 1.6: Substitution methods: linear substitutions, homogeneous DEs, Bernoulli DEs.

Week 4: Section 1.6 (cont'd): Exact DEs: definition of exactness, criterion of exactness, method of solution of exact DEs. Reducible 2nd order DEs: 1) variable y missing; 2) variable x missing.

Section 3.1: Homogeneous 2nd order linear DEs. The Superposition Principle.

Week 5: Section 3.1 (cont'd): The Existence/Uniqueness Theorem for 2nd order linear IVPs. Linear independence of two functions f and g; the Wronskian $W(f, g)$. The nature of the general solution of a 2nd order linear homogeneous DE. Solving IVPs. Solving 2nd order linear homogeneous DEs with constant coefficients; characteristic equation: (a) two real roots, (b) a double root and (c) non-real roots.

Week 6: Section 3.2: General solutions of n-th order linear DEs: Superposition Principle, Existence/Uniqueness for linear IVPs. Linear independence, Wronskian $W(y_1, \ldots, y_n)$. The nature of the solutions of an n-th order linear homogeneous DE.

Section 3.3: Homogeneous linear DEs with constant coefficients, characteristic equation: (a) distinct real roots, (b) repeated real roots, (c) complex roots, (d) repeated complex roots.

Week 7: Section 3.2 (end): The nature of solutions non-homogeneous linear DEs. Terminology: particular solution, complementary solution, associated homogeneous equation.

Section 3.5: Solving non-homogeneous linear DEs: Method of undetermined coefficients. Rule 1. The case of duplication and Rule 2.

Section 3.4: Application: Mechanical Vibrations: the free undamped spring;

Week 8: Section 3.4 (cont'd): Mechanical Vibrations: the free damped spring (overdamped, critically damped, underdamped systems).

Section 3.6: Mechanical Vibrations: Forced systems (free and damped), phenomenon of beats.

Chapter 7: Laplace transforms. Section 7.1: Definition, computing the Laplace transform of e^{at} and of t^a. Insert: the Gamma function.

Week 9: Section 7.1 (cont'd): Linearity of Laplace transforms (Theorem 1). Existence of Laplace transforms for piecewise continuous functions of exponential order (Theorem 2). Corollary. $\lim_{s\to\infty} F(s) = 0$. Uniqueness of inverse Laplace transforms (Theorem 3).

Section 7.2: Laplace transforms of derivatives (Theorem 1). Corollary: Laplace transforms of higher derivatives. Laplace transforms of IVPs.

Section 7.3: Computing inverse Laplace transforms: (a) The first translation theorem (Theorem 1); (b) The method of partial fractions. Rule 1: powers of a linear factor. Solving IVPs using Laplace transforms.

Week 10: Section 7.3 (cont'd): Method of completing the square. Partial fractions: Rule 2: powers of a quadratic factor.

Section 7.5: Piecewise continuous input functions. The unit step function $u(t)$. The second translation theorem (Theorem 1). Solving IVPs with step functions as forcing functions.

Chapter 5: Systems of first order linear DEs. Section 5.1: Matrix form of a system of linear first order DEs, solution of such a system. Homogeneous systems. Superposition principle for homogeneous systems (Theorem 1).

Week 11: Linear independence of (vector-valued) functions, Wronskians. The Wronskian criterion (Theorem 2). The general solution of linear system (Theorem 3). Initial value problems (IVPs) for 1st order systems. Solving such IVP's. The Eigenvalue Method I (Theorem 2). (This method is applicable when there exits a basis of eigenvectors of $\mathbb{R}^n$.)

Section 5.2: The Eigenvalue Method (for linear systems with constant coefficients). Theorem 1: Solutions via eigenvalues. Review of eigenvalues and of eigenvalues. Characteristic polynomial of a matrix. Method for finding eigenvalues and eigenvectors. Row reduction of matrices and back-substitution.

Week 12: Section 5.2 (cont'd): The Eigenvalue Method I (cont'd): repeated eigenvalues. The Eigenvalue Method II: Complex eigenvalues (Theorem 3). Application: Multiple tank mixing (cascading tanks).