# The Number of Digits

**Recall:** a computer stores and works with numbers in their *binary form*, i.e., to the base 2.

**More generally:** Given any base $b \in \mathbb{N}$, $b \geq 2$, we can write a given natural number $n \in \mathbb{N}$ in the form

(1) $$n = d_{k-1}b^{k-1} + \ldots + d_1 b + b_0,$$

with $d_{k-1} \neq 0$ and $0 \leq d_i < b$, for all $0 \leq i < k$. We then write

$$n = (d_{k-1}, \ldots, d_0)_b,$$

and call this the *representation of $n$ to the base $b$*.

**Remark:** We observe that $n$ satisfies the inequalities

(2) $$b^{k-1} \leq n \leq b^k,$$

which uniquely determine $k = \#\text{digits}_b(n)$, the *number of digits of $n$ to the base $b$*. Explictly:

(3) $$k = [\log_b(n)] + 1 = \left[\frac{\log(n)}{\log(b)}\right] + 1,$$

where $[x] = \text{floor}(x)$ denotes the greatest integer $n \leq x$. Thus:

$$\#\text{digits}_b(n) = O(\log(n)), \quad \text{if } n \geq 2.$$