Material: Data processing, AEP and entropy rates.

Readings: Sections 3.1 and 3.2 of the textbook.

The referred problems are from the textbook.

(1) Problem 2.11.
(2) Problem 2.20.
(3) Problem 3.1.
(4) Problem 3.3.
(5) Problem 3.10.

(6) Let \( \{(X_i, Y_i)\}_{i=1}^{\infty} \) be a two-dimensional discrete memoryless source with alphabet \( \mathcal{X} \times \mathcal{Y} \) and common distribution \( P_{XY} \). Find the limit as \( n \to \infty \) of the random variable

\[
\frac{1}{n} \log_2 \frac{P_{X^nY^n}(X^n, Y^n)}{P_{X^n}(X^n)P_{Y^n}(Y^n)}.
\]

(7) Binary Markov Source: Consider the binary homogeneous Markov source: \( \{X_n\}_{n=1}^{\infty} \), \( X_n \in \mathcal{X} = \{0, 1\} \), with

\[
\Pr\{X_{n+1} = j|X_n = i\} = \begin{cases} 
\frac{\rho}{1+\delta} & \text{if } i = 0 \text{ and } j = 1 \\
\frac{\rho+\delta}{1+\delta} & \text{if } i = 1 \text{ and } j = 1 
\end{cases},
\]

where \( n \geq 1 \), \( 0 \leq \rho \leq 1 \) and \( \delta \geq 0 \).

(a) Find the initial state distribution \( \Pr\{X_1 = 0\}, \Pr\{X_1 = 1\} \) required to make the source \( \{X_n\} \) stationary.

Assume in the next questions that the source is stationary.
(b) Find the entropy rate of \( \{X_n\} \) in terms of \( \rho \) and \( \delta \).

(c) If \( \delta = 1 \) and \( \rho = 1/2 \), compute the source redundancies \( \rho_D \), \( \rho_M \) and \( \rho_T \).

(d) Suppose that \( \rho = 1 \). Is \( \{X_n\} \) irreducible? What is the value of the entropy rate in this case?

(e) If \( \delta = 0 \), show that \( \{X_n\} \) is a discrete memoryless source and compute its entropy rate in terms of \( \rho \).

Additional Problems for MATH 874 students:

(8) Problem 3.2.

(9) Problem 3.4.

(10) Problem 3.20.