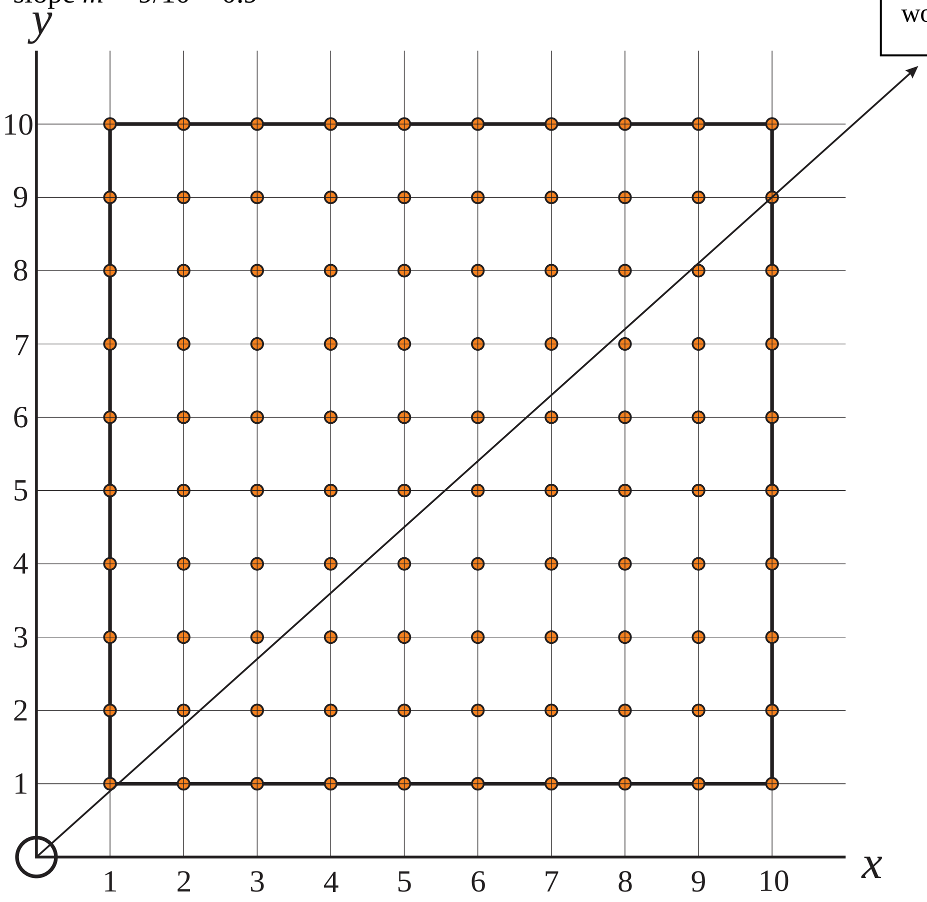
Neutrinos

The diagram below depicts a set of 100 atoms centred at the lattice points of a 10×10 grid. Each atom is circular with radius 1/10. A neutrino is shot from the origin (the point (0, 0)) and it follows a straight line through the grid. The equation of this line will have the form $y = mx$ where m is its slope. The path depicted in the diagram below has slope $m = 9/10 = 0.9$

I constructed this activity for the grade 9 curriculum which focuses a lot on lines and slopes. But it also provides an interesting argument that can be made either with similar triangles or with triangle trig. So it might work better for Grade 10.



We are interested in the general question of whether a neutrino whose path has a given slope m intersects any atoms, and if so which ones does it hit? For example the path $m = 9/10$ shown in the diagram passes right through the centre of the atom at (10, 9). It might also intersect the atoms at (1,1) and (9, 8), but that's hard to tell from the diagram.

Indeed our objective in this problem will be to find a precise mathematical condition for the path of a neutrino to hit any given atom.

By the way, neutrinos are so small that for this analysis we will assume they have zero size. Thus we are really asking whether the line of slope m passes within distance 1/10 of the lattice point at the centre of any atom.

We will work both with this 10×10 grid as well as with a 100×100 grid with proportionately smaller atoms.

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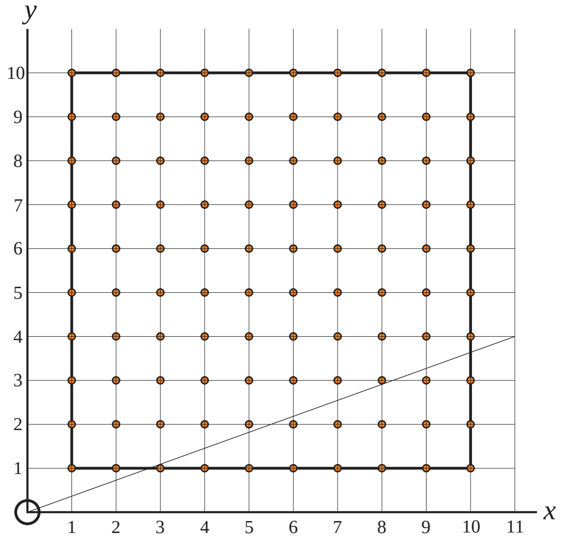
Example 1. Here we continue to work with the 10×10 grid with atoms of radius 1/10. We have drawn the path of a neutrino with slope $m = 4/11$.



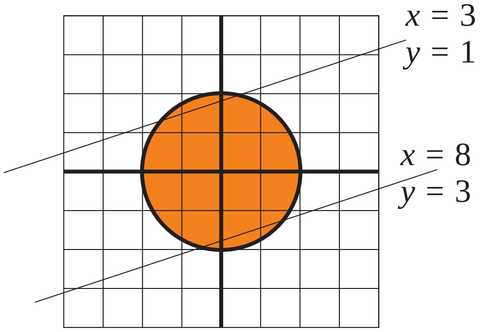
Observe that there are two atoms that the path might or might not hit—it's hard to judge from the diagram. Make the calculations to decide whether the path of the neutrino intersects these atoms. In each case plot as accurately as you can the trajectory of the neutrino on the diagram below, being careful with both the slope and the height of each path. Beside each line record the *x* and *y*-coordinates of the centre of the atom that it intersects.

Here we have to decide whether the neutrino intersects the atoms at (3, 1) and (8, 3). A simple thing to look at is the height of the line $y = mx$ at $x = 3$ and $x = 8$.

$x = 3$:

$y = mx = (4/11)(3) = 12/11 = 1 + 1/11 = 1.091$

We see that the *y*-value is 0.091 above the centre of the atom. Since the radius of the atom is 0.10, the path of the neutrino is just inside the atom at $x = 3$. To get the slope right I calculate the value of *x* when $y = 1$:
$x = y/m = 1/(4/11) = 11/4 = 2.75$



$x = 8$:

$y = mx = (4/11)(8) = 32/11 = 1 – 1/11 = 2.909$

We see that the *y*-value is 0.091 *below* the centre of the atom. Since the radius of the atom is 0.10, the path of the neutrino is just inside the atom at $x = 8$. The line has the same slope as the first and in fact is obtained by reflecting the first in the origin. [Why is that?]

There is a symmetry between the two lines. Can you see why this is expected?

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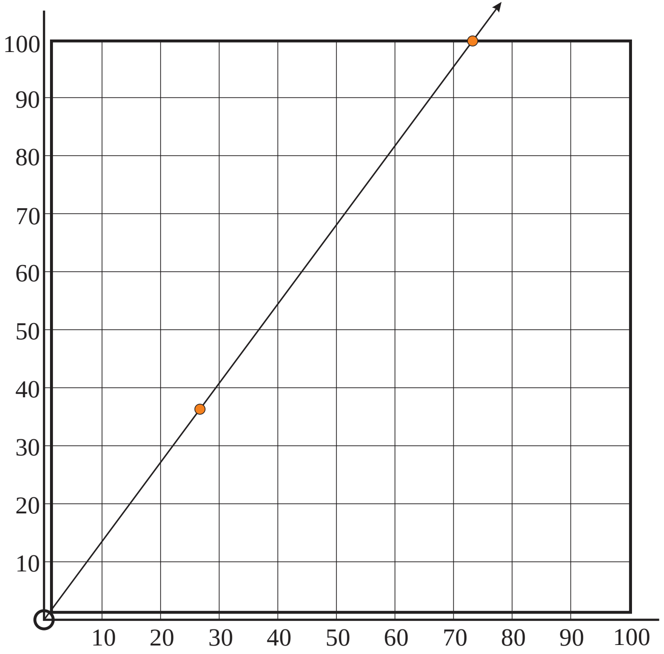
Example 2. Now we work with the 100×100 grid with atoms of radius 1/100 = 0.01. Suppose the path of the neutrino has slope $m = 100/73$—this is approximately 1.370.

(a) Draw the path of the neutrino on the diagram below. The path passes right through the centre of one of the atoms. Mark its location on the path.



It is quite likely that the neutrino intersects other atoms as well. How are we to determine this? Have a look at the data at the right, generated with a spreadsheet. What it gives us is the y-value of the intersection of the path with all vertical lines $x = x_0$ for integer values of x_0. See any intersections? What we want to look for are y-values that are close to an integer. But how close?

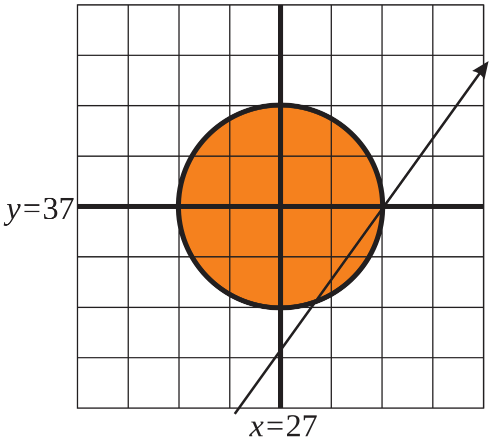
Well the closest entry in the table (other than the last one) is at $x = 27$, $y = 36.986$. The y-value is close to 37, but is it close enough?

(b) On the diagram at the right, draw the path of the neutrino through the atom. Make your drawing as accurate as possible. The best way to do that is to calculate the place where the path intersects the vertical line $x = 27$ and the horizontal line $y = 37$. That gives you two points that lie on the path. Does the path intersect the atom?

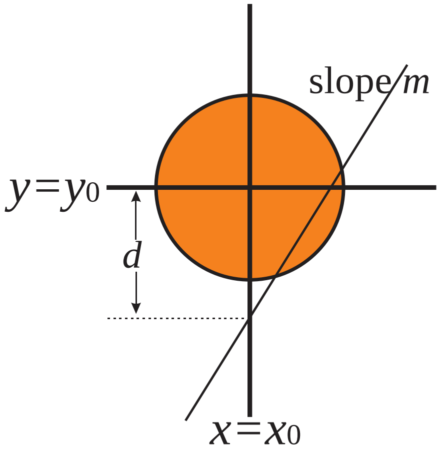
$x = 27$: $y = mx = (100/73)(27) = 36.986 = 37 - 0.014$
$y = 37$: $x = y/m = 37(73/100) = 27.010 = 27 + 0.010$

The path clearly intersects the atom.

$y = (100/73)x$			
x	y	x	y
1	1.370	38	52.055
2	2.740	39	53.425
3	4.110	40	54.795
4	5.479	41	56.164
5	6.849	42	57.534
6	8.219	43	58.904
7	9.589	44	60.274
8	10.959	45	61.644
9	12.329	46	63.014
10	13.699	47	64.384
11	15.068	48	65.753
12	16.438	49	67.123
13	17.808	50	68.493
14	19.178	51	69.863
15	20.548	52	71.233
16	21.918	53	72.603
17	23.288	54	73.973
18	24.658	55	75.342
19	26.027	56	76.712
20	27.397	57	78.082
21	28.767	58	79.452
22	30.137	59	80.822
23	31.507	60	82.192
24	32.877	61	83.562
25	34.247	62	84.932
26	35.616	63	86.301
27	36.986	64	87.671
28	38.356	65	89.041
29	39.726	66	90.411
30	41.096	67	91.781
31	42.466	68	93.151
32	43.836	69	94.521
33	45.205	70	95.890
34	46.575	71	97.260
35	47.945	72	98.630
36	49.315	73	100.000
37	50.685		

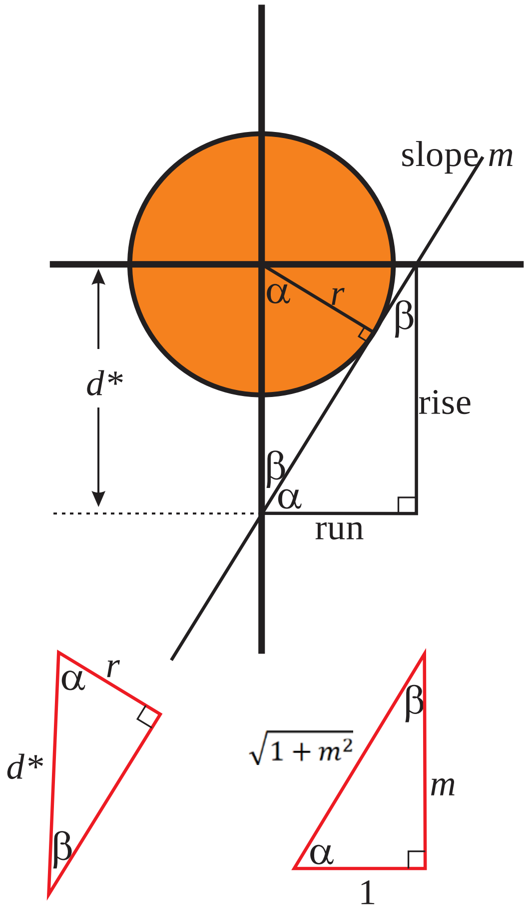


A precise mathematical condition.
Your task here is to find a precise mathematical condition for an intersection to occur. Work with general parameters––that is, find the condition for a neutrino path with slope *m* to intersect an atom of radius *r* centred at (x_0, y_0) .



There are many ways to formulate this condition. Here's a strategy using the diagram at the right. Calculate the distance *d* between the centre of the atom and the point where the path of the neutrino intersects the vertical line $x = x_0$. For *d* small enough, the path will intersect the atom; for *d* too large, it won't. Your job is to find the critical value *d** such that we will have an intersection exactly when *d* is less than (or equal to) *d**.

Well it should be clear that the critical configuration occurs when the path is tangent to the atom. Working with the diagram below right, find a formula for *d** in terms of *m* and *r*.



Solution
Most students will fasten attention on the red triangle on the left below the main diagram. It contains both *d** and *r*, and the problem is how to relate these to the slope *m* of the path. With some thought they might make the "standard" rise/run diagram that is the red triangle on the left. While we don't know all the sides of this triangle we know its *relative* side lengths to be 1, *m* and $\sqrt{1+m^2}$.

Now there are two ways to argue. The grade 9 students might notice that the two triangles are similar. They are both right-angled and the two angles β in the main diagram are clearly equal as they are created when the path crosses the two parallel vertical lines.

That gives them:

$$\frac{d^*}{r} = \frac{\sqrt{1+m^2}}{1}$$

and thus

$$d^* = r\sqrt{1+m^2}$$

This is the formula for the critical value of *d*.

The grade 10 students seem to prefer to use trig. Focusing on the angle β, they get

$$\sin\beta = \frac{r}{d^*} = \frac{1}{\sqrt{1+m^2}}$$

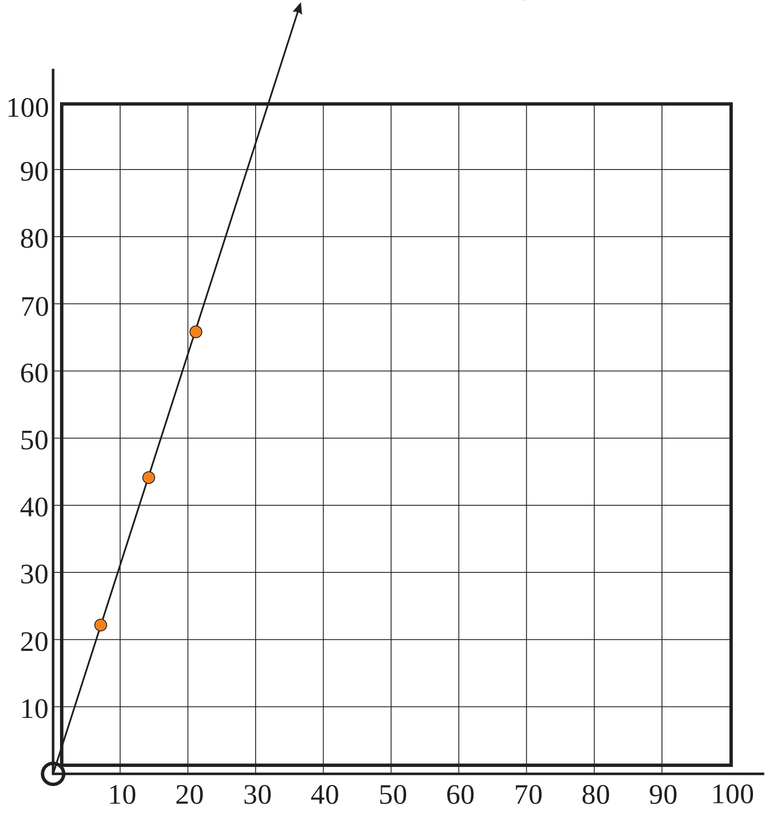
and the *d**-formula follows from there.

Example 3. The π-neutrino

Again we work with the 100×100 grid with atoms of radius 1/100 = 0.01, and we suppose the path of the neutrino has slope $m = \pi \sim 3.14159$.

x	y
1	3.142
2	6.283
3	9.425
4	12.566
5	15.708
6	18.850
7	21.991
8	25.133
9	28.274
10	31.416
11	34.558
12	37.699
13	40.841
14	43.982
15	47.124
16	50.265
17	53.407
18	56.549
19	59.690
20	62.832
21	65.973
22	69.115
23	72.257
24	75.398
25	78.540
26	81.681
27	84.823
28	87.965
29	91.106
30	94.248
31	97.389
32	100.531

(a) Draw the path of the neutrino on the diagram below.



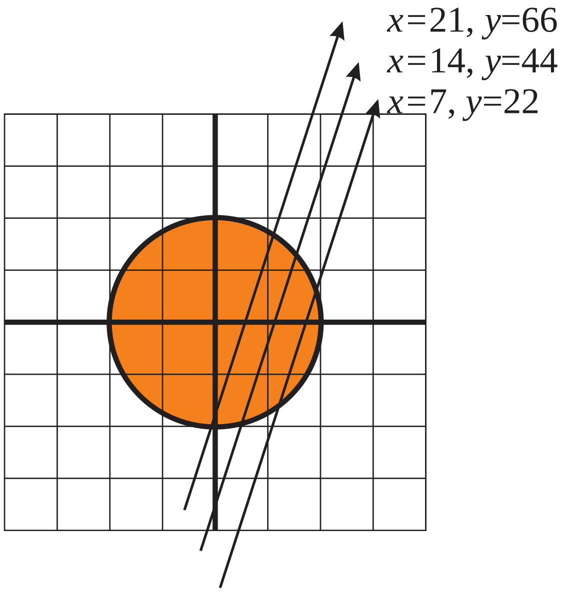
(b) Use your formula to calculate the critical d^* for this neutrino.

We have $r = 0.01$ and $m = \pi \sim 3.1416$. Then

$$d^* = r\sqrt{1+m^2} == 0.0303$$

(c) In the table at the right, circle all the (x, y) entries that correspond to an intersection. Mark these points on the path diagram above.

(d) On the diagram below right, draw the paths of the neutrino through all the different atoms that it intersects. Make your drawing as accurate as possible.

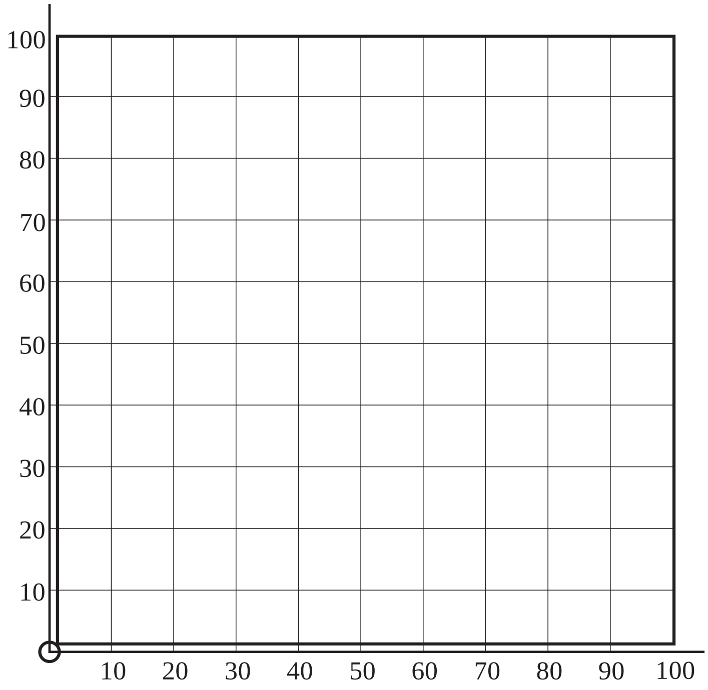


Problems

1. Again we work again with the 100×100 grid with atoms of radius 1/100 = 0.01. Suppose the path of the neutrino has slope $m = 341/213$—this is approximately 1.601.

$y = (341/213)x$				
x	y		x	y
1	1.60094		38	52.83099
2	3.20188		39	54.43192
3	4.80282		40	56.03286
4	6.40376		41	57.63380
5	8.00469		42	59.23474
6	9.60563		43	60.83568
7	11.20657		44	62.43662
8	12.80751		45	64.03756
9	14.40845		46	65.63850
10	16.00939		47	67.23944
11	17.61033		48	68.84038
12	19.21127		49	70.44131
13	20.81221		50	72.04225
14	22.41315		51	73.64319
15	24.01408		52	75.24413
16	25.61502		53	76.84507
17	27.21596		54	78.44601
18	28.81690		55	80.04695
19	30.41784		56	81.64789
20	32.01878		57	83.24883
21	33.61972		58	84.84977
22	35.22066		59	86.45070
23	36.82160		60	88.05164
24	38.42254		61	89.65258
25	40.02347		62	91.25352
26	41.62441		63	92.85446
27	43.22535		64	94.45540
28	44.82629		65	96.05634
29	46.42723		66	97.65728
30	48.02817		67	99.25822
31	49.62911		68	100.85915
32	51.23005			

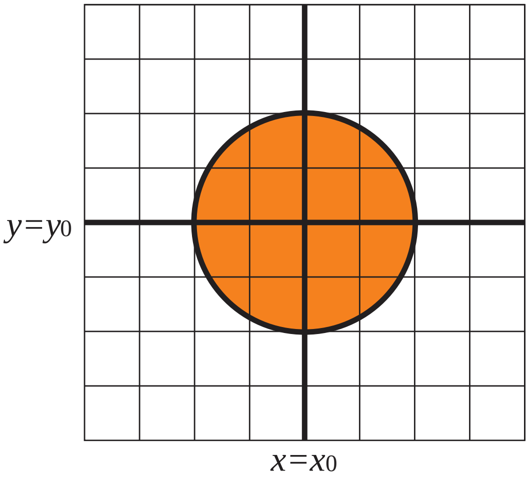
(a) Draw the path of the neutrino on the diagram below.



(b) Use your formula to calculate the value of d^*.

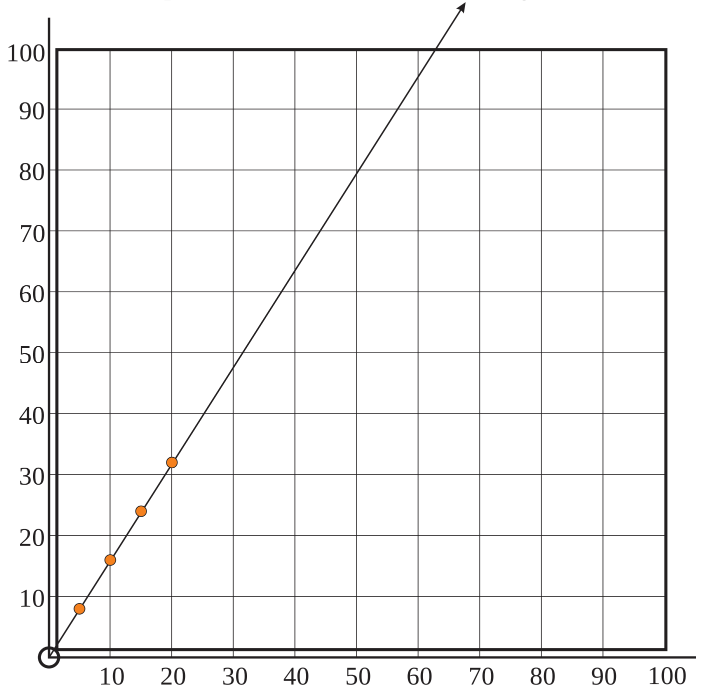
(c) In the table at the right, circle all the (x, y) entries that correspond to an intersection. Mark these points on the path diagram above.

(d) On the diagram at the right, draw the paths of the neutrino through all the different atoms that it intersects. atom. Make your drawing as accurate as possible. Again, to get the slope right, a good strategy is to plot the intercept of the path with the lines $x = x_0$ and $y = y_0$—at least for the first case. Discuss interesting features of the family of lines that you have drawn.



Solution to Problem 1. Again we work again with the 100×100 grid with atoms of radius 1/100 = 0.01. Suppose the path of the neutrino has slope $m = 341/213$—this is approximately 1.601.

(a) Draw the path of the neutrino on the diagram below.



(b) Use your formula to calculate the value of d^*.
We have $r = 0.01$ and $m = {}^{341}\!/_{213}$. Then

$$d^* = r\sqrt{1+m^2} = 0.0189$$

(c) In the table at the right, circle all the (x, y) entries that correspond to an intersection. Mark these points on the path diagram above.

(d) On the diagram at the right, draw the paths of the neutrino through all the different atoms that it intersects. atom. Make your drawing as accurate as possible. Again, to get the slope right, a good strategy is to plot the intercept of the path with the lines $x = x_0$ and $y = y_0$––at least for the first case. Discuss interesting features of the family of lines that you have drawn.

It is worth noting that the lines are parallel as they all have the same slope. They are also equally spaced as the x- and y-increases are always the same as we go from one line to the next, being

$$\Delta y = 5m - 8 \qquad \text{and} \qquad \Delta x = \frac{\Delta y}{m} = 5 - 8/m.$$

$y = (341/213)x$			
x	y	x	y
1	1.60094	38	52.83099
2	3.20188	39	54.43192
3	4.80282	40	56.03286
4	6.40376	41	57.63380
5	8.00469	42	59.23474
6	9.60563	43	60.83568
7	11.20657	44	62.43662
8	12.80751	45	64.03756
9	14.40845	46	65.63850
10	16.00939	47	67.23944
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13	20.81221	50	72.04225
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16	25.61502	53	76.84507
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24	38.42254	61	89.65258
25	40.02347	62	91.25352
26	41.62441	63	92.85446
27	43.22535	64	94.45540
28	44.82629	65	96.05634
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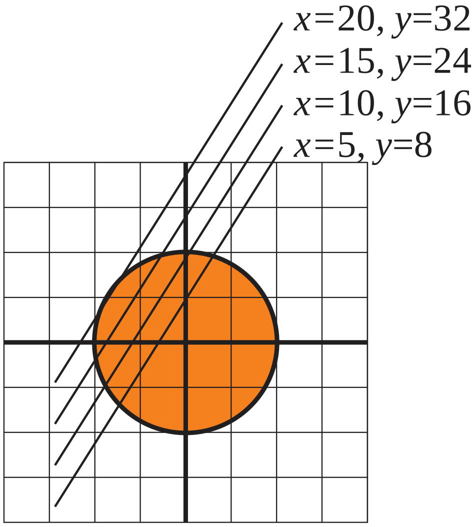


Neutrinos

Start by bringing your students into the incredible world of neutrinos, particularly since the recent Nobel-prize research work has taken place here at the SNOLAB in Sudbury.

https://en.wikipedia.org/wiki/Neutrino

A **neutrino** (/nuːˈtriːnoʊ/ or /njuːˈtriːnoʊ/) (denoted by the Greek letter ν) is a fermion (an elementary particle with half-integer spin) that interacts only via the weak subatomic force and gravity. The mass of the neutrino is much smaller than that of the other known elementary particles.

The neutrino is so named because it is electrically neutral and because its rest mass is so small (*-ino*) that it was originally thought to be zero. The weak force has a very short range, gravity is extremely weak on the subatomic scale, and neutrinos, as leptons, do not participate in the strong interaction. Thus, neutrinos typically pass through normal matter unimpeded and undetected.

Weak interactions create neutrinos in one of three leptonic flavors: electron neutrinos (*ν*e), muon neutrinos (νμ), or tau neutrinos (ντ), in association with the corresponding charged lepton. Although neutrinos were long believed to be massless, it is now known that there are three discrete neutrino masses with different tiny values, but they do not correspond uniquely to the three flavors. A neutrino created with a specific flavor is in an associated specific quantum superposition of all three mass states. Although only differences of squares of the three mass values are known as of 2016, cosmological observations imply that the sum of the three masses must be less than one millionth that of the electron.

Neutrinos oscillate between different flavors in flight. For example, an electron neutrino produced in a beta decay reaction may interact in a distant detector as a muon or tau neutrino. This oscillation occurs because the three mass state components of the produced flavor travel at slightly different speeds, so that their quantum mechanical wave packets develop relative phase shifts that change how they combine to produce a varying superposition of three flavors.

For each neutrino, there also exists a corresponding antiparticle, called an *antineutrino*, which also has half-integer spin and no electric charge. They are distinguished from the neutrinos by having opposite signs of lepton number and chirality. To conserve total lepton number, in nuclear beta decay, electron neutrinos appear together with only positrons (anti-electrons) or electron-antineutrinos, and electron antineutrinos with electrons or electron neutrinos.

Neutrinos can be created in several ways, including in beta decay of atomic nuclei or hadrons, nuclear reactions such as those that take place in the core of a star, and supernova, and when accelerated particle beams or cosmic rays hit atoms. The majority of neutrinos in the vicinity of the Earth are from nuclear reactions in the Sun. In the vicinity of the Earth, about 65 billion (6.5×10^{10}) solar neutrinos per second pass through every square centimeter perpendicular to the direction of the Sun. Neutrinos can be created artificially with nuclear reactors and particle accelerators.

There is intense research activity involving neutrinos, with goals that include the determination of the three neutrino mass values, the measurement of the degree of CP violation in the leptonic sector, which, if large enough, could provide a long-sought explanation (leptogenesis) for the excess of matter over anti-matter in the universe, and searches for evidence of physics beyond the Standard Model of particle physics such as neutrinoless double beta decay, which would be evidence for violation of lepton number conservation. Neutrinos are also finding practical applications, such as tomography of the interior of the earth.

The SNOLAB in Sudbury https://www.snolab.ca/

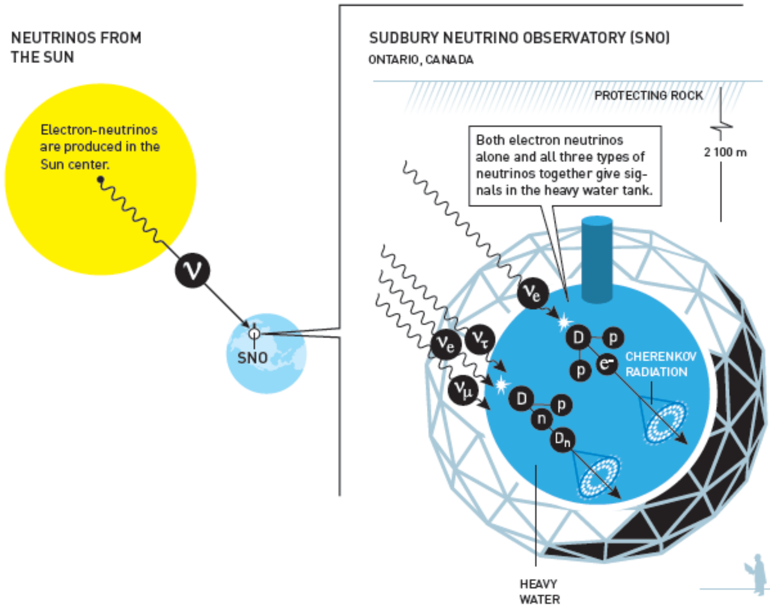
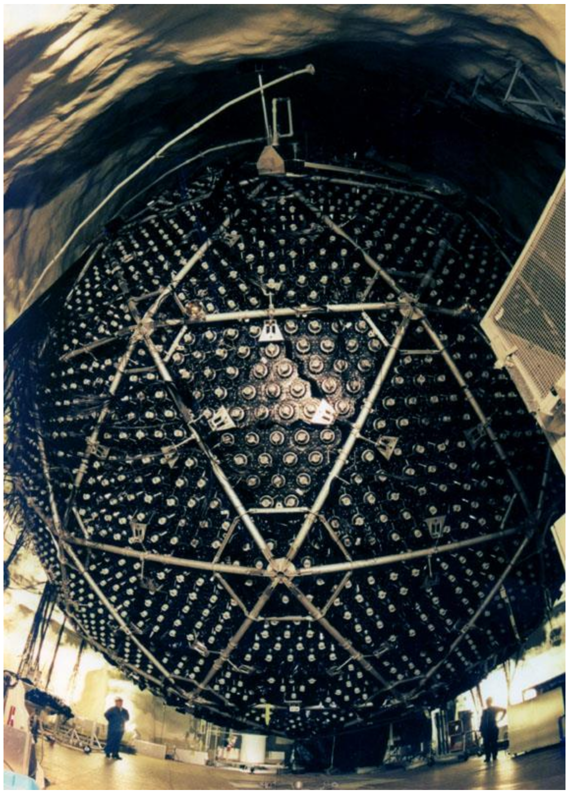
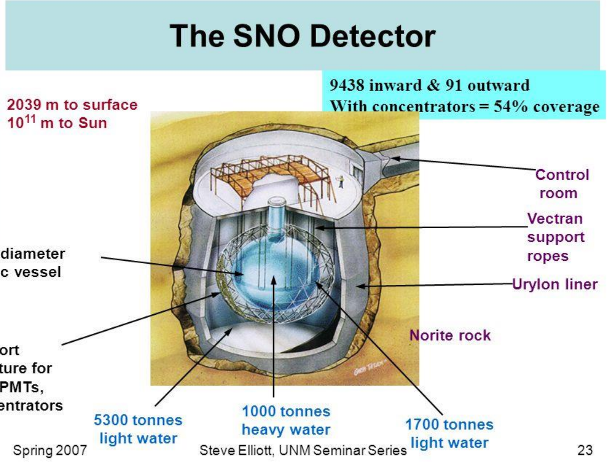


Illustration: © Johan Jarnestad/The Royal Swedish Academy of Sciences





The SNO Detector

9438 inward & 91 outward
With concentrators = 54% coverage

2039 m to surface
10^{11} m to Sun

Control room

Vectran support ropes

12 m diameter acrylic vessel

Urylon liner

Norite rock

Support structure for 9500 PMTs, concentrators

5300 tonnes light water

1000 tonnes heavy water

1700 tonnes light water

Spring 2007 Steve Elliott, UNM Seminar Series 23

SNO Data Taking and Discoveries The SNO detector is located in a huge rock cavern – 22 meters in diameter and 30 meters high, 2 kilometers underground at the Inco Creighton Mine, 20 minutes west of Sudbury Ontario. The rock above the detector filters out Cosmic Rays which would otherwise produce light photons in the detector, masking the tiny signals from neutrinos (seen only about 15 times per day).

Neutrinos (symbol ν - Greek letter nu) are detected in the clear plastic spherical tank at the centre of the detector, filled with 1000 tonnes of Heavy Water borrowed from AECL with the cooperation of Ontario Power Generation. Three types of reactions for neutrinos are seen in the heavy water which consists of D2O or deuterium oxide instead of the normal H2O, where D stands for deuterium - a heavy form of hydrogen with a proton (p) and an additional neutron (n) in its core (d). Each of these three reactions releases characteristic light photons (see the diagrams at right) which are detected by the 9600 ultrasensitive phototubes, surrounding the plastic tank. The outer part of the detector is filled with ultrapure normal water, which supports the central plastic tank and shields the core against tiny amounts of radioactivity in the surrounding rock. Any dust from the mine which enters the detector water can interfere with neutrino measurements, so SNO has been operated as a cleanroom laboratory from the time of its construction. Ultrapure materials are also used throughout the detector to minimize interferences.

In close to 7 years of data taking beginning in 1999, SNO has seen thousands of neutrinos through all three reactions. Careful modeling of each neutrino “event” allows the reaction type, energy, location and neutrino direction to be found with high probability. By comparing neutrino reaction rates, SNO announced in May 2001 that it had strong evidence that neutrinos (which start out as electron neutrinos in the Sun’s core) oscillate from this original type to other types as they travel from the sun to the detector. Two thirds of the neutrinos from the sun have changed from their original type to one of the other two types. Since these other neutrinos could not be identified by earlier experiments, SNO’s total neutrino numbers are much higher than those reported earlier, and now are in excellent agreement with the predictions of solar theories – the Solar Neutrino Problem has been solved. SNO’s papers detailing these results were the most cited papers in the world wide Physics literature in 2002 with more than 1000 references each.

The SNO experiment completed its data-taking phases in November 2006 and final analyses are approaching completion in 2009. The detector is being converted to the SNO+ experiment which will measure lower energy solar and geo neutrinos and search for rare nuclear decays to obtain more information about neutrinos.