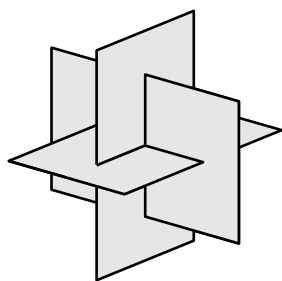


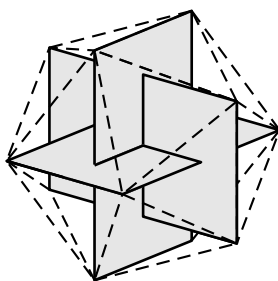
1. The *icosahedron* is one of the most beautiful solids discovered by the ancient Greeks. It has twelve vertices, thirty edges, and twenty faces made up of equilateral triangles. Imagining it without a drawing is a bit difficult, and part of Euclid's *Elements* is devoted to justifying that it even exists.

Let  $\vec{v}$  be a vector connecting the center of the icosahedron to one of the vertices, and  $\vec{w}$  be a vector connecting the center to any one of the five vertices near the one you picked for  $\vec{v}$ . Find:

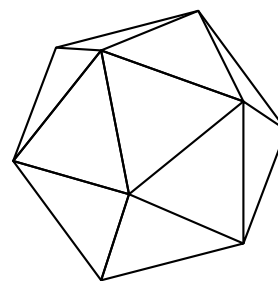
- An expression for  $\cos \theta$ , where  $\theta$  is the angle between  $\vec{v}$  and  $\vec{w}$ , and
- The value of  $\theta$  in degrees to at least three decimal places.



1



2

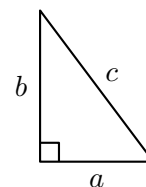


3

It might help to know the following construction of the icosahedron, due to Luca Pacioli, a friend of Leonardo da Vinci's, which first appeared in the 1509 book *De divina proportione*: Take three golden rectangles (i.e., rectangles whose proportions of side lengths are 1 to  $(1 + \sqrt{5})/2$ ) and interlock them to form the picture in the first figure. Then the twelve vertices of these rectangles form the twelve vertices of the icosahedron.

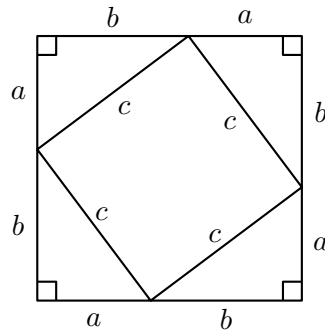
In part (a) of the question, give an algebraic expression for  $\cos \theta$  (which will involve  $\sqrt{5}$ 's) rather than a real number with decimals.

2. To complete our proof of the formula for the dot product, I'd like to prove one of the last things we used: the Pythagorean theorem. So, starting with a right angled triangle with sides of length  $a$ ,  $b$ , and  $c$ , (with  $c$  being the one opposite the right angle), like the one at right, we want to prove that  $a^2 + b^2 = c^2$ .



Here is the most elegant and famous proof of that fact. Arrange four copies of the triangle above so that they fit together to form a square of side length  $a + b$ . The

middle of this big square will contain a smaller (twisted) square of side length  $c$ , as in the diagram below:



Now, calculate the area of the big square in two ways. First, by using the fact that its a square of side length  $a + b$ , and second, by adding the area of the square in the middle to the area of the four triangles. Compare the two answers to prove the Pythagorean theorem.

**Reminder:** It is fine (and you are encouraged) to talk about the problems with other students, but the written homework you hand in must be your own. As in Math 120, you should acknowledge the assistance of any books, software, websites, students, or professors you used in completing the homework assignment.