Math 110 — Homework Assignment 4

DUE DATE: OCT. 14, 2016

1. Find the ranks of the following matrices.

(a) $\begin{bmatrix} 2 & 1 & 2 & 5 \\ 1 & 0 & 4 & 1 \\ 2 & 1 & 3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 4 & 3 \\ 5 & 1 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 1 & -1 & 6 \\ 2 & -1 & 5 & -1 & 7 \\ -1 & 1 & -4 & 1 & -3 \\ 0 & 1 & -3 & 1 & 1 \end{bmatrix}$

(d) Suppose the matrices in (a), (b), and (c) above are the coefficient matrices of a system of linear equations. Assuming that each of the systems has solutions, how many free parameters must the general solution to each of the systems have?

2. In each of the following three cases, write the vector $\vec{w}$ as a linear combination of the others.

(a) $\vec{w} = (1, 2)$; $\vec{v}_1 = (3, 5)$ and $\vec{v}_2 = (4, 7)$.

(b) $\vec{w} = (1, 1)$; $\vec{v}_1 = (1, 2)$, $\vec{v}_2 = (3, 4)$.

(c) $\vec{w} = (9, 5, 22)$; $\vec{v}_1 = (3, 2, 7)$, $\vec{v}_2 = (2, 1, 5)$, and $\vec{v}_3 = (0, 1, -1)$.

3.

(a) Write $\vec{w}_1 = (1, 0)$, $\vec{w}_2 = (0, 1)$, and $\vec{w}_3 = (11, 6)$ as linear combinations of $\vec{v}_1 = (5, 4)$ and $\vec{v}_2 = (2, 3)$.

(b) Suppose that T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$, and that

$$T(5, 4) = (6, -9, -9) \text{ and } T(2, 3) = (1, 2, 5).$$

Find

$$T(1, 0),\ T(0, 1) \text{ and } T(11, 6),$$

and explain how you know that T has to do this.