1. Compute each of the following vectors.

(a) $\begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & -7 & -4 \\ 1 & 5 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 3 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & -4 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix}$

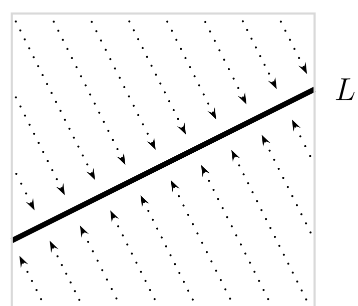
2. Find the standard matrix for each of the following linear transformations.

(a) The linear transformation $T\colon \mathbb{R}^3 \longrightarrow \mathbb{R}$ given by $T(\vec{v}) = \vec{u} \cdot \vec{v}$, where $\vec{u} = (2, 4, 3)$.

(b) The linear transformation $T\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ from given by $T(\vec{v}) = \vec{u} \times \vec{v}$ where $\vec{u} = (u_1, u_2, u_3)$. (This is the linear transformation that appeared in **H5**, Question 3.)

(c) The linear transformation $T\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ which is "rotate by $\pi/2$ counterclockwise around the z-axis". (Here "counterclockwise" means if you are on the positive z-axis looking down at the xy-plane, you want to rotate it counterclockwise.)

3. Given a line L in $\mathbb{R}^2$, *projection onto* L is the function $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ which sends every point in $\mathbb{R}^2$ to the nearest point on L, as shown in the diagram at right.



For any real number m, let $T_m : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the projection onto the line with slope m through the origin. This turns out to be a linear transformation, something you can assume when doing the question.

(a) Find the standard matrix for T_m, and explain your steps.

(b) As $m \to \infty$, what happens to the line of slope m? What happens to the matrix associated to T_m? Does this make sense?

Suggestion for (a): Start by finding two vectors where it is easy to understand the result of applying T_m, and then use linear combinations to deduce what T_m does to $\vec{e}_1$ and $\vec{e}_2$. (Another possibility : Use the projection formula.)

4. In this problem we will finish the proof of the theorem from the class of Thursday, October 20th.

Let A be an $m \times n$ matrix, and define a function $T\colon \mathbb{R}^n \longrightarrow \mathbb{R}^m$ by the rule $T(\vec{v}) = A\vec{v}$ for each $\vec{v} \in \mathbb{R}^n$. We want to show that T is a linear transformation.

In order to prove this, it will help to explicitly write out what "$A\vec{v}$" means. Let $\vec{w}_1$, $\vec{w}_2$, $\ldots$, $\vec{w}_n$ be the column vectors of A. Then for $\vec{v} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, $A\vec{v}$ is the vector $x_1\vec{w}_1 + x_2\vec{w}_2 + \cdots + x_n\vec{w}_n$ in $\mathbb{R}^m$.

Therefore, the issue really is : Show that the function $T\colon \mathbb{R}^n \longrightarrow \mathbb{R}^m$ defined by the rule

$$T(x_1, x_2, \ldots, x_n) = x_1\vec{w}_1 + x_2\vec{w}_2 + \cdots + x_n\vec{w}_n$$

is a linear transformation.

Your mission in this question : Show it!